$$\hat{\alpha} = \frac{-2}{\ln\frac{25}{40} + \ln\frac{25}{65} + \ln\frac{25}{100}} = \frac{-2}{-2.8118} = \boxed{0.7113}$$

single parameter Pareto(θ=?)

78) easy way

Math 404 Final Spring 2016 Name____________________

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hard way

1) An insurance policy has a deductible of 25 and an maximum payment per loss of 75. The three insurer loss payments under this policy were 15, 40, and 75. The insurer is only aware of those losses on which a payment is made. Assume the losses (not payments) were generated by a distribution with pdf $f(x) = \alpha x^{-(\alpha+1)}$ and cdf $F(x) = 1 - x^{-(\alpha)}$ where $x > 1$. Find the MLE $\hat{\alpha}$ of α.

$X_i = Y_i^P + d = 40, 65, 100$ ← censored

$$L(\alpha) = \frac{\prod f(x_i)\,[1-F(100)]}{(1-F(25))^3}$$

$$L(\alpha) = \frac{\alpha\, 40^{-(\alpha+1)}\ \alpha (65)^{-(\alpha+1)}\ 100^{-\alpha}}{(25^{-\alpha})^3} = \frac{\alpha^2 (2600)^{-(\alpha+1)}\, 100^{-\alpha}}{25^{-3\alpha}}$$

$$= \alpha^2 (40)^{-(\alpha+1)} (65)^{-(\alpha+1)}\, 100^{-\alpha}\, 25^{3\alpha}$$

$$\ln L(\alpha) = 2\ln(\alpha) - (\alpha+1)\ln(40) - (\alpha+1)\ln(65) - \alpha\ln(100) + 3\alpha\ln(25)$$

$$\frac{\partial \ln L(\alpha)}{\partial\alpha} = \frac{2}{\alpha} - \ln 40 - \ln 65 - \ln 100 + 3\ln 25 \overset{\text{set}}{=} 0$$

$$\frac{2}{\alpha} = 2.8118 \qquad \hat{\alpha} = \frac{2}{2.8118} = \boxed{0.7113}$$

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2) Suppose that the full-credibility standard is defined such that aggregate claim dollars will be within 5% of its true value 90% of the time. The number of claims follows a Poisson distribution, and the claim severity distribution X has $CV(X) = 2$. Find the indicated full credibility standard in terms of number of members insured (not number of claims), using limited fluctuation credibility.

Poisson(.5)

$$W = S \qquad e = e_F \qquad n_0 = \left(\frac{z_p}{k}\right)^2 = \left(\frac{1.645}{.05}\right)^2 = 1082.41$$

$$\text{answer} = \frac{n_0}{\lambda}\left(1 + [CV(X)]^2\right) = \frac{1082.41}{0.5}(1+4)$$

$$= \boxed{10824.1}$$

$VaR_p(X) = \theta\,[-\ln(p)]^{1/\tau} = -\theta\,[\ln(p)]^{-1}$

Percentile

weibull had 117.1 and 13.9

3) Suppose $X \sim$ Inverse Weibull$(\theta = 100, \tau = 1)$. Simulate two values of x_i from this distribution if $u_1 = 0.69$ and $u_2 = 0.13$. Hint: $x_i = VaR_{u_i}(X)$.

$x_i = -\theta\,[\ln(u_i)]^{-1}$ so $x_1 = -100\,[\ln(.69)]^{-1} = \boxed{269.4955}$

$x_2 = -100\,[\ln(.13)]^{-1} = \boxed{49.0143}$

Q11#9

25 e 4) Suppose you are estimating the mean θ of losses with the MLE $\overline{X}$ assuming an exponential (θ) distribution.

actual losses 1, 2, 5, 10, 50: $\overline{X} = 13.6$

Q11#3

bootstrap samples:

2, 10, 1, 2, 2: $\overline{X} = 3.4$

50, 10, 50, 2, 2: $\overline{X} = 22.8$

10, 50, 2, 1, 1: $\overline{X} = 12.8$

5, 2, 5, 1, 50: $\overline{X} =$? 12.6

a) Compute $\overline{X}$ for the fourth bootstrap sample, and find $\widehat{MSE}$.

$$\widehat{MSE} = \frac{1}{B}\sum_{i=1}^{B}(T_i^* - T)^2 = \frac{(3.4-13.6)^2 + (22.8-13.6)^2 + (12.8-13.6)^2 + (12.6-13.6)^2}{4}$$

$$= \frac{190.32}{4} = 47.58$$

or $\widehat{MSE} = \frac{\bar{x}^2}{n} = \frac{\frac{1}{4}(184.04)}{5} = 85.26$

b) Find $\widehat{bias} = \frac{1}{B}\sum_{i=1}^{B}(T_i^* - T) = \left(\frac{1}{B}\sum_{i=1}^{B}T_i^*\right) - T$.

$$= \frac{3.4+22.8+12.8+12.6}{4} - 13.6$$

$$= 12.9 - 13.6 = \boxed{-0.7}$$

This page no longer on final see Quiz 11 1),2)

50

$E[X] = \frac{\theta}{\alpha-1}$ $\quad E X^2 = \theta^2 \frac{\Gamma(3)\Gamma(\alpha-2)}{\Gamma(\alpha)} = \frac{2\theta^2}{(\alpha-1)(\alpha-2)} = V(X)+(E(X))^2$

e 5) The following claim data were generated from a Pareto distribution: 130, 20, 350, 218, 1822. Then $\sum X_i = 2540$ and $\sum X_i^2 = 3,507,008$. Find the method of moments estimators of α and θ ~~then estimate the limited expected value $E(X \wedge 500)$ at $d = 500$.~~

$m = \bar{X} = \frac{2540}{5} = 508, \quad \frac{1}{5}\sum X^2 = \frac{3507008}{5} = 701401.6 = t$

$\frac{\theta}{\alpha-1} \stackrel{set}{=} 508, \quad \theta = 508(\alpha-1) \qquad \frac{2\theta}{\alpha-2} = 1380.7118$

$\frac{2\theta^2}{(\alpha-1)(\alpha-2)} \stackrel{set}{=} 701401.6 \qquad \frac{\alpha-1}{\alpha-2} = \frac{1380.7118}{2(508)} = 1.3590$

$\hat{\alpha} - 1 = 1.359\hat{\alpha} - 2(1.3590)$

$0.359\hat{\alpha} = 2(1.359) - 1$

$$\boxed{\begin{aligned}\hat{\alpha} &= 4.7853 \\ \hat{\theta} &= 508(\hat{\alpha}-1) = 1922.952\end{aligned}}$$

easy way 23 iv)

$\hat{\alpha} = \frac{2(t-m^2)}{t-2m^2} = \frac{2(t-m^2)}{185273.6} = 4.7858$

and $\hat{\theta} = \frac{mt}{t-2m^2} = \frac{mt}{185273.6} = 1923.1662$

6) The prior distribution for Θ has pdf $\pi(\theta) = \frac{1}{\theta^2}$ for $\theta > 1$. Claim sizes $X|\Theta = \theta \sim$ Pareto($\alpha = 2, \theta$). A claim of 2 is observed. Find the posterior pdf. Note that the support is $\theta > 1$.

$\pi(\theta|x) \propto f(x|\theta)\,\pi(\theta) = \frac{\alpha\theta^\alpha}{(\theta+x)^{\alpha+1}} \frac{1}{\theta^2} \propto (\theta+2)^{-3}$

$1 = c\int_1^\infty (\theta+2)^{-3}\,d\theta = c\left.\frac{(\theta+2)^{-2}}{-2}\right|_1^\infty = \frac{c}{2}3^{-2} = \frac{c}{18}$

So $\boxed{\pi(\theta|x) = 18(\theta+2)^{-3} = \frac{18}{(\theta+2)^3}, \quad \theta > 1}$

7) There was a random sample of 30 auto claims. Suppose the fitted distribution gave the following table where 0 parameters were estimated. Fill in the table and do a 4 step χ^2 test. E2.118

interval	$O_i = n_i$	p_i	$E_i = np_i$	$C_i = \dfrac{(O_i - E_i)^2}{E_i}$
0–500	3	0.27	8.1	3.2111 = 3.2111
500–2498	8	0.28	8.4	0.01905
2498–4876	9	0.26	7.8	0.1846
4876–infinity	10	0.19	5.7	3.2439
	30	1	30	6.6586

i) H_0 fitted dist is good H_1 not H_0

ii) $Q = 6.6586$

iii) $df = k - r - 1 = 4 - 0 - 1 = 3$ | $.95$: 7.815

$Q < 7.815$ fail to reject H_0

iv) the fitted dist is good

or not enough evidence to conclude that the fitted dist is not good

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8) The number of claims made by an individual insured in a year has a Poisson(λ) distribution where $\lambda \sim \text{gamma}(\alpha = 2, \theta = 1)$. Two claims are observed in year 1 and one claims is observed in year 2. Using Buhlmann credibility, estimate the number of claims in year 3. E3.115

14.2) $v = \alpha\theta = 2$, $a = \alpha\theta^2 = 2$, $k = \frac{1}{\theta} = 1$

$n = 2$, $X_1 = 2$, $X_2 = 1$ $\quad \bar{X} = \frac{2+1}{2} = 1.5$

$$E(X) = E[E(X|\lambda)] = E(\lambda) = \alpha\theta = 2$$

$$EPV = v = E[v(\lambda)] = E[V(X|\lambda)] = E(\lambda) = \alpha\theta = 2$$

$$VHM = a = V(\mu(\lambda)) = V[E(X|\lambda)] = V(\lambda) = \alpha\theta^2 = 2$$

$$k = \frac{v}{a} = \frac{2}{2} = 1, \quad z = \frac{n}{n+k} = \frac{2}{2+1} = \frac{2}{3}$$

$$P = z\bar{X} + (1-z)E(X) = E(X) + z[\bar{X} - E(X)]$$

$$= 2 + \frac{2}{3}(1.5 - 2) = 2 - \frac{1}{3} = \boxed{\frac{5}{3} = 1.6667}$$

9) The table is used for a sample of 5. Use the 4 step Kolmogorov Smirnov test for whether the data are from the distribution with cdf $F(x) = 1 - \frac{1}{(1+x)^4}$. e 3d18

x_i	$F_n(x_i)$	$F_n(x_i-)$	$F^*(x_i)$	$\max(\|F_n(x_i) - F^*(x_i)\|, \|F_n(x_i-) - F^*(x_i)\|)$	
0.2	1/5	0/5	0.518	0.518	
0.7	2/5	1/5	0.880	0.680	$\Leftarrow D$
0.9	3/5	2/5	0.923	0.523	
1.1	4/5	3/5	0.949	0.349	
1.3	5/5	4/5	0.964	0.164	

i) Ho fitted dist is good H_1 not Ho

ii) $D = 0.680$

iii) critical value $= \frac{1.36}{\sqrt{n}} = \frac{1.36}{\sqrt{5}} = 0.608 < D = .680$

reject Ho

iv) the fitted model is not good

10) Suppose $X_1, ..., X_n$ are iid EXP(θ). The MLE of θ is $\hat{\theta} = \overline{X}$. Find the asymptotic variance of the MLE of $g(\theta) = \ln(\theta)$. E2d18

$$V(\hat{\theta}) = \frac{V(X)}{n} = \frac{1}{n I_1(\theta)} = \frac{\theta^2}{n} = Var(\hat{\theta}) \quad \leftarrow 739$$

$$\underset{\text{asy var}}{Var}(g(\hat{\theta})) = [g'(\theta)]^2 \, var(\hat{\theta})$$

$$g(\theta) = \ln(\theta), \quad g'(\theta) = \frac{1}{\theta}$$

$$Var(g(\hat{\theta})) = \frac{1}{\theta^2} \frac{\theta^2}{n} = \boxed{\frac{1}{n}}$$

$f(x|\theta) = \frac{1}{\theta} e^{-x/\theta}$, $\ln f(x|\theta) = -\ln\theta - \frac{x}{\theta}$, $\frac{d \ln f(x|\theta)}{d\theta} = \frac{-1}{\theta} + \frac{x}{\theta^2}$

$\frac{d^2 \ln f(x|\theta)}{d\theta^2} = \frac{1}{\theta^2} - \frac{2x}{\theta^3}$ $I_1(\theta) = E\left(\frac{-1}{\theta^2} + \frac{2x}{\theta^3}\right) = \frac{-1}{\theta^2} + \frac{2}{\theta^2} = \frac{1}{\theta^2}$ hard way

11) Members of four classes of insured can have 0 or 1 claims, with the following probabilities. 211018

	Number of claims		w $\mu(\theta)$	$v(\theta)$
class	0	1		
I	0.9	0.1	0.1	0.09
II	0.8	0.2	0.2	0.16
III	0.5	0.5	0.5	0.25
IV	0.1	0.9	0.9	0.09

$= (1-0)^2 \cdot .8(.2)$ Bernoulli Shortcut

A class is chosen at random, with probability 0.25, and four insureds are selected from this class. The total number of claims from the four insureds is two. (Get $\overline{X}$ from this information.) so selected with prob 1/4

If five insureds are selected at random from the same class, estimate their total number of claims using Buhlmann Straub credibility, that is determine P, the Buhlmann Straub credibility estimate of the total number of claims for 5 randomly selected insureds. (You need to compute $\mu(\theta)$ and $v(\theta)$ for class II.) $n=4$

$$\overline{X} = \frac{2}{4} = 0.5 \quad \text{claims per person}$$

$$\mu = E[\mu(\Theta)] = \frac{1}{4}(.1 + .2 + .5 + .9) = 0.425$$

$$v = E[v(\Theta)] = \frac{1}{4}(0.09 + 0.16 + 0.25 + 0.09) = 0.1475 = \frac{.59}{4}$$

$$a = V[\mu(\Theta)] = E w^2 - \mu^2 = \frac{1}{4}\left[(.1)^2 + (.2)^2 + (.5)^2 + (.9)^2\right] - (.425)^2$$

$$= \frac{1.11}{4} - (0.425)^2 = 0.096875$$

$$k = \frac{v}{a} = \frac{.1475}{.096875} = 1.5226$$

$$Z = \frac{n}{n+k} = \frac{4}{4+1.5226} = 0.7243 = \frac{na}{na+v} = \frac{.3875}{.535}$$

$$P_c = 5[\mu + Z(\overline{X} - \mu)] = 5[0.425 + 0.7243(0.5 - 0.425)]$$

$$= 5(0.4793)$$

$$= \boxed{2.3966}$$

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