

Use Exam review p 1-2

Math 404 Quiz 1 Spring 2018
YOU ARE BEING GRADED FOR WORK

Name _____

- 1) Suppose $X \sim \text{Pareto}(\alpha, \theta)$. If $\hat{\alpha} = 2$ and $\hat{\theta} = 400$, estimate the following quantities.
a) $E(Y^P) = e_X(200)$ if there is a deductible of 200.

$$e_X(200) = \frac{\theta + d}{\alpha - 1} \approx \frac{400 + 200}{2 - 1} = \boxed{600}$$

$$\begin{aligned} \text{b) } VaR_{0.5}(X) &= \pi_{0.5} = \theta \left[(1-p)^{-\frac{1}{\alpha}} - 1 \right] \approx 400 \left[(0.5)^{-\frac{1}{2}} - 1 \right] \\ &= 400(0.4142) = \boxed{165.6854} \end{aligned}$$

$$\text{c) } E(X \wedge 200) = \frac{\theta}{\alpha - 1} \left[1 - \left(\frac{\theta}{x + \theta} \right)^{\alpha - 1} \right] \approx$$

$$400 \left[1 - \frac{400}{200 + 400} \right] = 400 \left(1 - \frac{4}{6} \right) = \frac{400}{3} = \boxed{133.3333}$$

- 2) Estimate $VaR_{0.99}$ for the single parameter Pareto(α, θ) distribution if $\hat{\alpha} = 3$, and $\hat{\theta} = 20$.

$$= \theta \left[(1-p)^{-\frac{1}{\alpha}} \right] \approx 20 \left[(0.01)^{-\frac{1}{3}} \right] =$$

$$\boxed{92.8318}$$

3) Suppose $X \sim \text{exponential}(\theta)$. If $\hat{\theta} = 500$, estimate the loss elimination ratio $\text{LER}(120)$ if there is a deductible of 120.

$$= 1 - e^{-\frac{d}{\theta}} \approx 1 - e^{-\frac{120}{500}} = 1 - e^{-0.24} = \boxed{0.2134}$$

$$\uparrow$$

$$= \frac{E(x-d)}{E(x)} = \frac{\theta(1 - e^{-d/\theta})}{\theta}$$

4) Suppose $X \sim \text{EXP}(\theta)$ with $\hat{\theta} = 50.5$. Estimate the following (for $x > 0$):

a) $\hat{f}(x) = \frac{1}{\theta} e^{-x/\theta} = \boxed{\frac{1}{50.5} e^{-x/50.5}}$

b) $\hat{S}(x) = e^{-x/\theta} = \boxed{e^{-x/50.5}}$

c) $\widehat{E(X)} = \hat{\theta} = \boxed{50.5}$

5) Suppose $N \sim \text{Poisson}(\lambda)$ with $\bar{N} = \hat{\lambda} = 6.5$. Estimate $P(N = 4)$.

$$P_k = \frac{e^{-\lambda} \lambda^k}{k!}$$

$$P_4 \approx \frac{e^{-6.5} (6.5)^4}{4!} = \frac{e^{-6.5} (6.5)^4}{24}$$

$$= \boxed{0.1182}$$