table 51.5 $v = \alpha\theta$, $k = \frac{1}{\theta}$
(142) $a = \alpha\theta^2$

Math 404 Quiz 10 Spring 2018 Name____________________
YOU ARE BEING GRADED FOR WORK

1) The number of claims X made by an individual insured in a year has a Poisson(λ) distribution where $\lambda \sim$ gamma($\alpha = 1, \theta = 1.2$). Three claims are observed in year 1 and no claims are observed in year 2. Note that $n = 2, X_1 = 3, X_2 = 0, X|\lambda \sim Poisson(\lambda)$ and $\overline{X} = 3/2$.

a) Find $\mu(\lambda) = E(X|\lambda)$. $= \boxed{\lambda}$

b) Find $v(\lambda) = V(X|\lambda)$. $= \boxed{\lambda}$

7 c) Find $v = EPV = E[V(X|\lambda)]$. $= E(\lambda) = \alpha\theta = \boxed{1.2}$

7 d) Find $a = VHM = V[E(X|\lambda)]$. $= V(\lambda) = \alpha\theta^2 = \boxed{1.44}$

7 e) Find $k = EPV/VHM = v/a$. $= \frac{1.2}{(1.2)^2} = \frac{1}{1.2} = \boxed{0.8333} = \frac{5}{6}$

7 f) Find $E(X) = E[E(X|\lambda)]$.

$= E(\lambda) = \boxed{1.2}$

7 g) Find $Z = \frac{n}{n+k}$. $= \frac{2}{2+\frac{1}{1.2}} = \frac{2}{2.8333} = \boxed{0.7059}$

7 h) Using Buhlmann credibility, estimate the number of claims in year 3: see (127)

$P = E(X) + Z(\overline{X} - E(X))$.

$= Z\overline{X} + (1-Z)E(X)$

$= 1.2 + 0.7059\,(1.5 - 1.2) = \boxed{1.4118}$

$\left(= 0.7059\ \frac{3}{2} + (1 - 0.7059)\,1.2 \right.$
$\left. = 0.7059(1.5) + 0.2941(1.2) \right)$

$\overline{50}$

$\overline{X} = \frac{3+0}{2}$ is given

$t = \frac{2100 + 4(750) + 9(100) + 16(50)}{8000} = \frac{6800}{8000} = 0.85$

total claims k	number of policies n_k
0	5000
1	2100
2	750
3	100
4	50

$\hat{\sigma}_E^2 = t - m^2 = 0.85 - (.5125)^2 = .5873$

$\hat{\sigma}_U^2 = \frac{n}{n-1}\hat{\sigma}_E^2 = \frac{8000}{7999}(.5873) = .5874$

2) During a Year 1, 8000 policies had the above claim experience. Assume the number of claims followed a conditional Poisson(λ) distribution. Assume $\hat{\sigma}_U^2 = 0.5874$. See 131) from the exam reviews. For tabled data let x_i be the claim total for the ith policy holder, let and n_k be the number of claims with claim total $= k$. Then $n = 8000 = \sum n_k$, $m = \overline{X} = \sum(kn_k)/n$, $t = \sum_{i=1}^n x_i^2/n = \sum k^2 n_k/n$.

a) Find $\overline{X} = \hat{v}$.

$= \frac{0 + 2100 + 750(2) + 100(3) + 50(4)}{8000}$

$= \frac{4100}{8000} = \frac{41}{80} = \boxed{0.5125}$

b) Find $\hat{a} = \hat{\sigma}_U^2 - \hat{v}$.

$= 0.5874 - 0.5125 = \boxed{0.0749}$

c) Find $\hat{Z}$.

$= \frac{1}{1 + \frac{\hat{v}}{\hat{a}}} = \frac{1}{1 + \frac{.5125}{.0749}} = \boxed{0.1275}$

$\hat{k} = 6.8425$

d) A randomly selected policyholder had 1 ($= X_i$) claim in Year 1. Determine the semiparametric empirical Bayes estimate of the number of claims in Year 2 for the same policyholder $P_C^1 = \overline{X} + \hat{Z}(X_i - \overline{X})$.

$0.5125 + 0.1275(1 - 0.5125) = \boxed{0.5747}$

$\overline{50}$