Math 404 Quiz 2 Spring 2018 Name____________________

YOU ARE BEING GRADED FOR WORK

1) Let X have cdf $F(x) = (1-\epsilon)F_{X_1}(x) + \epsilon F_{X_2}(x)$ where $0 \le \epsilon \le 1$, $X_1 \sim Poisson(\theta)$, and $X_2 \sim Pareto(5, \theta)$. Find the method of moments estimator of θ if ϵ **is known**.

$$E(X) = (1-\varepsilon)EX_1 + \varepsilon EX_2 = (1-\varepsilon)\theta + \varepsilon \frac{\theta}{4} \overset{set}{=} m$$

$$\theta\left(1-\varepsilon+\frac{\varepsilon}{4}\right) = \theta\left(1-\frac{3\varepsilon}{4}\right) = m$$

$$\boxed{\hat{\theta} = \frac{m}{1-\frac{3}{4}\varepsilon} = \frac{4m}{4-3\varepsilon}}$$

0.75

Use exam1 rev 24)

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e 2) If $X_1, ..., X_n$ are iid Gamma(α, θ), then the MMEs are $\hat{\alpha} = \dfrac{m^2}{t-m^2} = \dfrac{m^2}{\hat{\sigma}^2_E}$, $\hat{\theta} = \dfrac{\hat{\sigma}^2_E}{m} = \dfrac{t-m^2}{m}$. Suppose in a sample of size $n = 1000$, $\sum_{i=1}^{1000} X_i = 9955$ and $\sum_{i=1}^{1000} X_i^2 = 147214$. Find the method of moments estimators of α and θ.

$$m = \bar{x} = \frac{9955}{1000} = 9.955, \quad t = \frac{\sum x_i^2}{n} = 147.214$$

$$\hat{\sigma}_E^2 = t - m^2 = 48.1120$$

$$\hat{\theta} = \frac{t-m^2}{m} = \frac{147.214-(9.955)^2}{9.955} = \boxed{4.8329} = \frac{\hat{\sigma}_E^2}{m}$$

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$$\hat{\alpha} = \frac{m^2}{t-m^2} = \frac{(9.955)^2}{147.214-(9.955)^2} = \boxed{2.0598} = \frac{m^2}{\hat{\sigma}_E^2}$$

12

Use Exam1 rev 23i)

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use Exam1 rev 23 i)

3) The observations 5, 10, 25, 50, and 250 were obtained as a random sample from a Gamma distribution with unknown parameters α and θ. Determine the method of moments estimate of θ.

(a) $\hat{\theta} = 120$
(b) $120 < \hat{\theta} \leq 130$
(c) $130 < \hat{\theta} \leq 140$
(d) $140 < \hat{\theta} \leq 150$
(e) $150 < \hat{\theta}$

$$\hat{\theta} = \frac{\hat{\sigma}_E^2}{m} = \frac{t - m^2}{m}$$

$$m = \bar{x} = \frac{\Sigma x_i}{n} = \frac{340}{5} = 68$$

$$t = \frac{1}{n}\Sigma x_i^2 = \frac{65750}{5} = 13150$$

$$\hat{\theta} = \frac{13150 - (68)^2}{68} = \frac{8526}{68} = \boxed{125.3824}$$

25

4) For a sample of 8 losses, $\sum_{i=1}^{8} X_i = 396$ and $\sum_{i=1}^{8} X_i^2 = 53098$. Claims are assumed to follow a Pareto(α, θ) distribution. Estimate α and θ using the method of moments.

$$m = \frac{\Sigma x_i}{n} = \frac{396}{8} = 49.5, \quad t = \frac{\Sigma x_i^2}{n} = \frac{53098}{8} = 6637.25$$

$$\hat{\sigma}_E^2 = t - m^2 = 6637.25 - (49.5)^2 = 4187.0000$$

$$\hat{\alpha} = \frac{2(t - m^2)}{t - 2m^2} = \frac{2\hat{\sigma}_E^2}{t - 2m^2} = \frac{2(4187)}{6637.25 - 2(49.5)^2} = \frac{8374}{1736.75}$$

$$\boxed{\hat{\alpha} = 4.8216}$$ 12

$$\hat{\theta} = \frac{mt}{t - 2m^2} = \frac{49.5(6637.25)}{6637.25 - 2(49.5)^2} = \frac{328543.875}{1736.75} =$$

$$\boxed{\hat{\theta} = 189.1717}$$ 13

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use Exam1 rev 23 iv)