Math 404 Quiz 3 Spring 2018 Name____________________

YOU ARE BEING GRADED FOR WORK

1) You are given the following data:
1, 1, 2, 2, 4, 6, 6, 7, 9, 11, 12, 15, 17. Find the smoothed empirical estimate, $\hat{\pi}$, of the 70th percentile.

(a) $\hat{\pi} \leq 8.5$
(b) $8.5 < \hat{\pi} \leq 9.0$
(c) $9.0 < \hat{\pi} \leq 9.5$
(d) $9.5 < \hat{\pi} \leq 10$
(e) $10 < \hat{\pi}$

$n=13, \lfloor (n+1)p \rfloor = \lfloor 14(.7) \rfloor = \lfloor 9.8 \rfloor = 9 = j$

$h = 9.8 - 9 = 0.8$

$\hat{\pi}_{.7} = (1-h)\, X_{(j)} + h\, X_{(j+1)} =$

$0.2(9) + 0.8(11) = \boxed{10.6}$

30

2) Suppose losses follow an EXP(θ) distribution. Find θ by matching the 88th percentile using the table below.

interval (a,b]	proportion
(0,1000]	0.39
(1000,2000]	0.25
(2000,4000]	0.24
(4000,7500]	0.10
(7500,14000]	0.02

sum to 0.88

$\hat{\pi}_{.88} = 4000 \overset{set}{=} \pi_{.8} = VaR_p(X) = -\theta \ln(1-p)$

$$\hat{\theta} = \frac{4000}{-\ln(0.12)} = \boxed{1886.5579}$$

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$$L(P) = \prod_{i=1}^{n} f(x_i) = \prod_{i=1}^{3} f(x_i)$$

3) You are given the following three observations: 0.74 0.81 0.95. You fit a distribution with the following density function to the data: $f(x) = (p+1)x^p$, where $0 < x < 1$ and $p > -1$. Determine the maximum likelihood estimate of p.

(A) 4.0
(B) 4.1
(C) 4.2
(D) 4.3
(E) 4.4

$$L(P) = (P+1)^3 [.74(.81).95]^P$$

.5694

$$\ln L(P) = 3\ln(P+1) + P\ln[.74(.81).95]$$

$$\frac{d \ln L(P)}{dP} = \frac{3}{P+1} + \underbrace{\ln[.74(.81).95]}_{-.5631} \overset{\text{set}}{=} 0$$

$$3 = (P+1)\left[-\ln(.74(.81).95)\right] = (P+1)(0.5631)$$

$$\hat{P} = \frac{3}{0.5631} - 1 = \boxed{4.3275}$$

(See back of Pg 1, too $\hat{P} = \frac{-n}{\Sigma \ln x_i} - 1$)

10

4) You believe that a Poisson distribution, with a mean of λ, reflects the number of claims per policy each year. You observe how many claims occur under each of three policies during a year, and compile the following observation data:

Claims per Policy	Number of Observations
0 claims	2
1 claims	1

data 0, 0, 1

Calculate the maximum likelihood estimate of λ.

Hint: use the formula from the review or use $L(\lambda) = c\prod_{i=1}^{n} e^{-\lambda}\lambda^{x_i}$.

$$\hat{\lambda} = \bar{X} = \frac{2(0)+1(1)}{3} = \boxed{\frac{1}{3} = 0.3333}$$

hard way $L(\lambda) = c\lambda^0 e^{-\lambda}\lambda^0 e^{-\lambda}\lambda^1 e^{-\lambda} = c\lambda e^{-3\lambda}$

$\ln L(\lambda) = \ln(\lambda) - 3\lambda + d$

$\frac{d \ln L(\lambda)}{d\lambda} = \frac{1}{\lambda} - 3 \overset{\text{set}}{=} 0$ or

$3\lambda = 1$ so $\hat{\lambda} = \frac{1}{3}$

or $L(\lambda) = c\lambda^{\Sigma x_i} e^{-n\lambda}$

$\ln L(\lambda) = \Sigma x_i \ln \lambda - n\lambda + d$

$\frac{d \ln L(\lambda)}{d\lambda} = \frac{\Sigma x_i}{\lambda} - n \overset{\text{set}}{=} 0$

or $\Sigma x_i = n\lambda$ so $\hat{\lambda} = \bar{x} = \frac{\Sigma x_i}{n} = \frac{1}{3}$

30