Y^P are payments $\quad f(x) = \frac{1}{\theta} e^{-x/\theta}, \quad 1 - F(x) = e^{-x/\theta}$

X are losses

Math 404 Quiz 4 Spring 2018 Name____________________

YOU ARE BEING GRADED FOR WORK

1) An insurance policy has a deductible of 20 and a policy limit (maximum payment per loss) of 80. The four insurer loss payments under this policy have been 10, 30, 50, and 80. The insurer is only aware of those losses on which a payment is made. Assume that the ground–up losses were generated by an exponential distribution with a mean of θ.

a) You are given Y^P. Convert the Y^P values to X values that have been truncated and possibly censored using $X = Y^P + d$.

Y^P	10	30	50	80
X	30	50	70	100

add 20

censored

X is truncated at $d = 20$ and censored at $u = 100$

b) Find $L(\theta)$. =

$$\frac{f(30)f(50)f(70)[1-F(100)]}{[1-F(20)]^4} =$$

← all 4 are truncated

$$\frac{\frac{1}{\theta}e^{-\frac{30}{\theta}}\frac{1}{\theta}e^{-\frac{50}{\theta}}\frac{1}{\theta}e^{-\frac{70}{\theta}}e^{-\frac{100}{\theta}}}{(e^{-\frac{20}{\theta}})^4} = \frac{1}{\theta^3}e^{-(30+50+70+100-80)/\theta}$$

$$= \boxed{\frac{1}{\theta^3} e^{-\frac{170}{\theta}}}$$

c) Find the MLE of θ.

$$\text{easy way! } \hat{\theta} = \frac{\sum (x_i - d)}{m} = \frac{\sum Y_i^P}{m} = \frac{10+30+50+80}{3} = \frac{170}{3} =$$

m ← # uncensored

56.6667

hard way! $\ln L(\theta) = -3\ln\theta - \frac{170}{\theta}$

$$\frac{d \ln L(\theta)}{d\theta} = -\frac{3}{\theta} + \frac{170}{\theta^2} \stackrel{\text{set}}{=} 0 \quad \text{or} \quad 170 = 3\hat{\theta}$$

so $\hat{\theta} = \frac{170}{3}$

60

2) You are given: (i) The number of claims follows a Poisson distribution with mean λ. (ii) Observations other than 0 and 1 have been deleted from the data. (iii) The data contain an equal number of observations of 0 and 1. Determine the maximum likelihood estimate of λ. Hint: Let n be the number of observations that were not deleted. Then

$$L(\lambda) = \left(\frac{p_0}{F(1)}\right)^{n/2} \left(\frac{p_1}{F(1)}\right)^{n/2} = \left(\frac{e^{-\lambda}}{e^{-\lambda} + \lambda e^{-\lambda}}\right)^{n/2} \left(\frac{\lambda e^{-\lambda}}{e^{-\lambda} + \lambda e^{-\lambda}}\right)^{n/2} = \frac{\lambda^{n/2}}{(1+\lambda)^n}.$$

$$\ln L(\lambda) = \frac{n}{2} \ln(\lambda) - n \ln(1+\lambda)$$

$$\frac{d \ln L(\lambda)}{d\lambda} = \frac{n}{2\lambda} - \frac{n}{1+\lambda} \overset{set}{=} 0 \quad \text{or}$$

$$n(1+\lambda) = n\, 2\lambda \quad \text{or } 1+\lambda = 2\lambda$$

$$\text{or } \boxed{\hat{\lambda} = 1}$$

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3) You observe the following 4 ground up claims from a data set that is truncated below at 1000 (no losses below 1000 will be submitted to the insurance company).
1028.5334, 1445.7569, 1952.3840, 3529.0490

Find the MLE of θ if an EXP(θ) distribution is fit to the data. You can use $\hat{\theta} = \frac{1}{n}\sum_{i=1}^{n}(X_i - d)$ since $m = n$ for this data set (all $m = n$ cases are uncensored). d is given

$$= \frac{28.5334 + 445.7569 + 952.384 + 2529.0490}{4}$$

$$= \frac{3955.7233}{4} = \boxed{988.9308}$$

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(y^P are payments, x are losses)