Math 404 Quiz 5 Spring 2018 Name___________________

YOU ARE BEING GRADED FOR WORK

1) Suppose there is a deductible $d = 1000$, a maximum payment of 9000, and only the (positive) payments were observed. For these payments, the sample size $n = 807$ and 172 payments of 9000 were made. If the sum of the payments $\sum_{i=1}^{807} Y_i^P = 3235824$, and an Exponential($\theta$) distribution is fitted, find the MLE of θ. E2rev 78a)

$$\hat{\theta} = \frac{\Sigma Y_i^P}{\underset{\underset{807-172}{\uparrow}}{m}} = \frac{3235824}{635} = \boxed{5095.7858}$$

15

2) Suppose there is a deductible $d = 100$, a maximum payment of 200, and the losses (including the deductible) were observed, but there is no information about losses less than 100. For these observed losses, the sample size $n = 83$ of which 17 are censored at 300. If the sum of the squared losses $\sum_{i=1}^{83} X_i^2 = 4022458$, and a Weibull($\theta, \tau = 2$) distribution is fitted, find the MLE of θ. Hint: $\sum_{i=1}^{83}(X_i^2 - d^2) = (\sum_{i=1}^{83} X_i^2) - 83d^2$. E2rev 78b)

$$\hat{\theta} = \sqrt{\frac{\Sigma(x_i^2 - d_i^2)}{\underset{\underset{83-17}{\uparrow}}{m}}} = \sqrt{\frac{4022458 - 830000}{66}}$$

$$= \sqrt{\frac{3192458}{66}} = \sqrt{48370.5758}$$

$$= \boxed{219.9331}$$

15

e 3) Suppose $X \sim U(0, \theta)$ with 430 observations below 350 and 70 observations above 350. Find the MLE of θ. E2rev 79)

$(0, \overset{c}{350})$ 430 = m

$(350, \infty)$ 70
$\overset{\shortparallel}{F}$

500 = n

$$\hat{\theta} = \frac{500}{430}\, 350 = \boxed{406.9767}$$

$$\hat{\theta} = \frac{n}{m} c$$

20

E2 rev 73g) $\hat{\lambda} = \bar{X} = \frac{8+9+9+5+14}{5} = 9$

4) You fit an Poisson(λ) distribution to the following data: 8, 9, 9, 5, 14.
a) Find $Var(\hat{\lambda})$.

$\hat{\lambda} = \bar{X}$ so $Var(\hat{\lambda}) = Var(\bar{X}) = \frac{\sigma^2}{n} = \boxed{\frac{\lambda}{n}} = \frac{V(X)}{n}$

or $Var(\hat{\lambda}) = \frac{1}{nI_1(\lambda)} = \frac{1}{n\frac{1}{\lambda}} = \frac{\lambda}{n}$ $\left(\widehat{var}(\hat{\lambda}) = \frac{\hat{\lambda}}{n} = \frac{9}{5}\right)$

b) Find a 90% confidence interval for λ. E2 rev 77)

$\hat{\lambda} \pm 1.645\sqrt{\widehat{var}(\hat{\lambda})} = 9 \pm 1.645\sqrt{\frac{9}{5}}$ $\left(\frac{9}{5} = 1.8\right)$

$= 9 \pm 2.2070 = \boxed{(6.7930, 11.2070)}$

$\left(\hat{\theta} \pm z_p SE(\hat{\theta}),\ SE(\hat{\theta}) = \text{estimated } SD(\hat{\theta})\right)$

40

5) Suppose $X_1, ..., X_n$ are iid with $I_1(\theta) = \frac{2}{\theta^2}$ where $\theta > 0$. Find the asymptotic variance of the MLE of $\sqrt{\theta}$. E2 rev 70)

$\tau(\theta) = \theta^{\frac{1}{2}}$, $\tau'(\theta) = \frac{1}{2}\theta^{-\frac{1}{2}} = \frac{1}{2\sqrt{\theta}}$,

$[\tau'(\theta)]^2 = \frac{1}{4\theta}$. $Var(\hat{\theta}) = \frac{1}{nI_1(\theta)}$

$Var(\tau(\hat{\theta})) = [\tau'(\theta)]^2 Var(\hat{\theta}) = \frac{[\tau'(\theta)]^2}{nI_1(\theta)} = \frac{(\frac{1}{2}\theta^{-\frac{1}{2}})^2}{n\frac{2}{\theta^2}}$

$= \left(\frac{1}{4\theta}\right)\frac{1}{n\frac{2}{\theta^2}} = \boxed{\frac{\theta}{8n}}$

10