Math 404 Quiz 7 Spring 2018 Name____________________

YOU ARE BEING GRADED FOR WORK $95 = n$

1) You fit various loss models to 95 claim sizes using maximum likelihood. The fits maximizing the likelihood for a given number of parameters have the following loglikelihoods. $0.5\ln(95) \approx 2.2769$

	model	number of parameters	maximal loglikelihood	$\ln(L_r) - \frac{r}{2}\ln(95)$
L_1	1	1	−487	-489.2769
	2	1	−487.5 (bad models)	(-489.2769)
	3	2	−487 (bad models)	(-491.5538)
L_2	4	2	−484.1	$-488.6538 \leftarrow$ b)
L_3	5	3	−482	$-488.8308 = -482 - \frac{3}{2}\ln(95)$

a) Using the likelihood ratio algorithm (test) at 95% confidence (5% significance), how many parameters are in the selected model? Show how the selected model was chosen.

$2(\ln L_2 - \ln L_1) = 2(-484.1+487) = 5.8 > 3.84 = \chi^2_{1,.95}$ prefer 2 to 1

$2(\ln L_3 - \ln L_2) = 2(-482+484.1) = 4.2 > 3.84$ prefer 3 to 2

parameters in L_3 and L_2

$\boxed{\text{3 parameter model 5}}$

b) Using the Schwarz Bayesian Criterion, how many parameters are in the selected model? Hint: eliminate one of the 1 parameter models and one of the 2 parameter models by inspection.

$\boxed{\text{Model 4 with 2 parameters is selected}}$

(maximizes $\ln(L_r) - \frac{r}{2}\ln(n)$)

$F(x) = 1 - e^{-x/10}$

2) The table is used for a sample of 4 claim payments. Use the 4 step Kolmogorov Smirnov test for whether the claims are from an exponential(10) distribution.

x_i	$F_n(x_i)$	$F_n(x_i-)$	$F^*(x_i)$	$\max(\lvert F_n(x_i) - F^*(x_i)\rvert, \lvert F_n(x_i-) - F^*(x_i)\rvert)$	
2	1/4	0/4	0.181	0.181	$= .181 - 0$
5	2/4	1/4	0.393	0.143	$= .393 - .25$
13	3/4	2/4	0.727	0.227	$= .727 - .5 \leftarrow D$
22	4/4	3/4	0.889	0.139	$= .889 - .75$

i) H_0 EXP(10) is a good fit H_1 not H_0

ii) $D = 0.227$

iii) critical value $= \frac{1.36}{\sqrt{n}} = \frac{1.36}{2} = 0.68$

$D < 0.68$ fail to reject H_0

iv) the EXP(10) dist is a good fit

(or not enough evidence to conclude EXP(10) is not a good fit)

3) Suppose that the claim amount X is uniform$(0, \Theta)$ where the prior distribution of Θ has pdf $\pi(\theta) = \frac{500}{\theta^2}$ for $\theta > 500$. Two independent claims of 400 and 600 are observed. Find the pdf of the posterior distributions of Θ. Hint: $f(\boldsymbol{x}|\theta) = f(400, 600|\theta) = \frac{1}{\theta}\frac{1}{\theta}$ for $\theta > \max(600, 400) = 600$. Hence the support of the posterior pdf is $\theta > 600$.

$\pi(\theta|\underline{x}) \propto \pi(\theta)\, f(\underline{x}|\theta) \propto \theta^{-4}$ $\quad \theta > 600$

$\int_{600}^{\infty} c\theta^{-4}\,d\theta = c \left.\frac{\theta^{-3}}{-3}\right|_{600}^{\infty} = c\,\frac{1}{3(600)^3} = 1$

so $\boxed{\pi(\theta|\underline{x}) = 3(600)^3\theta^{-4} = 6.48(10^8)\,\theta^{-4} = 648{,}000{,}000\,\theta^{-4} \text{ for } \theta > 600}$