Math 404 Quiz 8 Spring 2018 Name____________________

YOU ARE BEING GRADED FOR WORK

$\alpha+1=4.7$ so $\alpha=3.7$

e 1) Suppose the posterior distribution $\pi(\theta|\boldsymbol{x}) \propto (19.3+\theta)^{-4.7}$ where $\theta > 0$. Find $E(\Theta|\boldsymbol{x})$.

Hint: recognize that the posterior distribution from 110) on the exam review.

$\Theta|\underset{\sim}{x} \sim \text{pareto}(\alpha = 3.7, \tilde{\theta} = 19.3)$

$$E(\Theta|x) = \frac{\tilde{\theta}}{\alpha-1} = \frac{19.3}{2.7} = \boxed{7.1481}$$

2) Suppose $\pi(1) = \pi(2) = 0.5$ is the prior distribution where Θ takes on values $\theta = 1, 2$. Suppose $n = 2$ and the likelihood (pdf) is

$$f(\boldsymbol{x}|\theta) = f(x_1, x_2|\theta) = \frac{1}{\theta^2} e^{-x_1/\theta} e^{-x_2/\theta}$$

for $x_1, x_2 > 0$. Find the posterior probability $\pi(1|\boldsymbol{x}) = \pi(1|x_1 = 1.4, x_2 = 0.3)$ using Bayes' theorem.

$$= \frac{f(\underset{\sim}{x}|1)\,\pi(1)}{f(\underset{\sim}{x}|1)\,\pi(1) + f(\underset{\sim}{x}|2)\,\pi(2)} = \frac{f(\underset{\sim}{x}|1)}{f(\underset{\sim}{x}|1) + f(\underset{\sim}{x}|2)}$$

$$= \frac{e^{-1.4}\,e^{-0.3}}{e^{-1.4}\,e^{-0.3} + \frac{1}{2^2} e^{-1.4/2}\,e^{-0.3/2}} = \frac{.1827}{.1827 + \underbrace{\frac{.4274}{4}}_{.10685}}$$

$$= \frac{.1827}{.28955} = \boxed{0.6310}$$

3) The prior distribution of Θ has pdf

$f(x|\theta) = \frac{\alpha\theta^\alpha}{(\theta+x)^{\alpha+1}} = \frac{2\theta^2}{(\theta+x)^3}$

$$\pi(\theta) = \frac{1}{\theta^2}$$

for $1 < \theta < \infty$. Given $\Theta = \theta$, claim sizes follow a Pareto($\alpha = 2, \theta$) distribution. A claim of 2 is observed.

a) Find the pdf $\pi(\theta|x = 2)$ of the posterior distribution.

$$\pi(\theta|2) \propto \pi(\theta) f(2|\theta) \propto \frac{1}{\theta^2} \frac{2\theta^2}{(\theta+x)^3} \overset{x=2}{\propto} (\theta+2)^{-3}$$

$$1 = c\int_1^\infty (\theta+2)^{-3} d\theta = c \left. \frac{(\theta+2)^{-2}}{-2} \right|_1^\infty = \frac{c}{2} 3^{-2} = \frac{c}{18}$$

so $\boxed{\pi(\theta|2) = 18(\theta+2)^{-3} = \frac{18}{(\theta+2)^3}, \ \theta > 1}$

b) Calculate the posterior probability that Θ exceeds 3.
Hint: calculate $\int_3^\infty \pi(\theta|x = 2)d\theta$.

$$\int_3^\infty 18(\theta+2)^{-3} d\theta = 18 \left. \frac{(\theta+2)^{-2}}{-2} \right|_3^\infty$$

$$= 0 + 9(5)^{-2} = \boxed{\frac{9}{25} = 0.36}$$