1] Get familiar with the distributions on p1–2 of the exam 1 review.

2] Know The <u>two parameter pareto distribution</u> $\equiv$ the <u>pareto distribution</u>.

The <u>single parameter pareto distribution</u> will use those four words.

3] pxvii A <u>loss</u> is a cash outflow for an insurance company made to pay a benefit for a claim.

An <u>actuarial loss model</u> is a model for the company's losses.

4] N = number of losses is a discrete Random Variable

X = severity of loss is usually a continuous RV

<u>usually $X \geq 0$</u>, $N \geq 0$ is an integer.

5] * The <u>cumulative distribution function</u> (cdf) $F(x) = P(X \leq x)$.

The survival function $S(x) = P(X > x)$ 1.5

$= 1 - F(x)$. $0 \le S(x) \le 1$, $S(x)$

is nonincreasing, right continuous,

$\lim_{x \to -\infty} S(x) = 1$, $\lim_{x \to \infty} S(x) = 0$.

If $X \ge 0$, $S(0) = 1$.

6] If $X \ge 0$ is a continuous RV, and

the pdf (prob density function) is $f(x)$, then the

hazard function $h(x) = \dfrac{f(x)}{S(x)}$, and

the cumulative hazard function

$$H(x) = \int_0^x h(t)\,dt = -\ln[S(x)]$$

* $F(x), S(x), f(x), h(x), H(x) \ge 0$ are nonnegative.

7] For a discrete RV X,

the pmf (prob mass function) $p(x) = f(x) = P(X = x)$

and $p_k = p(k) = P(X = k)$.

8] p13 The support of a RV =

$\{x : f(x) > 0\}$ or $\{x : p(x) > 0\}$, but

sometimes values of x where $f(x) = 0$ or $p(x) = 0$

are added.

In this class, if X is a loss severity RV, usually the support is a subset of $[0, \infty)$.

The support of N = number of claims is a subset of $\{0, 1, 2, \ldots\}$.

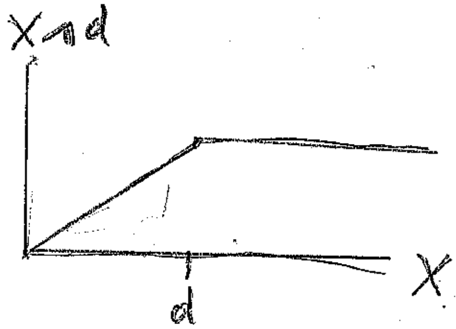
Often formulas for $F(x)$, $S(x)$, $f(x)$, $p(x)$ are only given for the support.

9] * $\min(X, d) = X \wedge d = t(X) = \begin{cases} X, & X \le d \\ d, & d < X \end{cases}$

is the limited loss RV and is right censored: values of $X > d$ are recorded as d. The limited expected value

$E(X \wedge d) = \int_0^d x f(x)\,dx + d\,S(d) = \int_0^d S(x)\,dx$. $E[X \wedge x]$

is given on E1 rev p 1-2 for some brand name RVs. (usually things are for the insurance company)



If a policyholder has a loss X and deductible d, the policy holder loses X if $X \le d$ (gets nothing) and loses d (gets $X-d$) if $X > d$.

So $E(X \wedge d)$ is the expected loss (per loss) for a policy holder with deductible d.

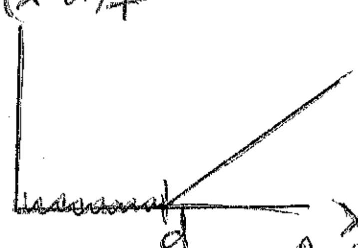
10] For $X \geq 0$, if $\lim_{x \to \infty} x\,S(x) = 0$, then (2.5)

$$E(X) = \int_0^\infty x f(x)\,dx \overset{\downarrow}{=} \int_0^\infty S(x)\,dx = \mu = \text{mean}.$$

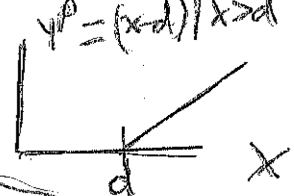
11] $\overset{\text{standard deviation}}{\sigma = \sqrt{V(X)}} = SD(X)$ and $\sigma^2 = V(X) =$ variance

12] Know The per loss RV $Y^L = t(X)$

$$= \underbrace{(X-d)_+}_{\text{positive part of } X-d} = \begin{cases} 0, & X \leq d \\ X-d, & X > d \end{cases}$$



Y^L is the "payment", possibly 0, made per loss.



13] Know The per payment RV Y^P = excess loss variable and $Y^P = (X-d) \mid X > d$. The mean excess loss function $e_X(d) = E(Y^P)$ is given for several distributions. Only positive payments are allowed.

14] $$E(Y^P) = e_X(d) = \frac{E(Y^L)}{S_X(d)} = \frac{E(X-d)_+}{S_X(d)} = \frac{E(X) - E(X \wedge d)}{S_X(d)}$$

15] $E(Y^L) = E(X-d)_+ = S_X(d)\,E(Y^P) = E(X) - E(X \wedge d) = \int_d^\infty S_X(x)\,dx$

16) discrete losses

$x_1 < x_2 \le \cdots \le x_m \le d < x_{m+1} < \cdots < x_k$ ($<$ or $\le$)

X

x	x_1	x_2	$\cdots$	x_m	x_{m+1}	$\cdots$	x_k
$P(X=x_i)$	p_1	p_2	$\cdots$	p_m	p_{m+1}	$\cdots$	p_k
$Y^L=(X-d)_+$ y	0				$x_{m+1}-d$	$\cdots$	x_k-d
$P(Y^L=y)$	$p_1+\cdots+p_m$				p_{m+1}	$\cdots$	p_k
Y^P y					$x_{m+1}-d$	$\cdots$	x_k-d
$P(Y^P=y)$					$\frac{p_{m+1}}{S_X(d)}$	$\cdots$	$\frac{p_k}{S_X(d)}$

$$S_X(d) = p_{m+1} + \cdots + p_k$$

ex) $d=1$

x	0	1	2	3
$P(X=x)$	0.1	0.2	0.3	0.4
Y^L	0		$2-1=$ 1	$3-1=$ 2
	$.3 = .1+.2$		.3	.4
Y^P			1	2
			$\frac{.3}{.7}$	$\frac{.4}{.7}$

$$EY^L = E(X-1)_+ = \Sigma\, y\, P(Y^L = y)$$

$$= 1(.3) + 2(.4) = \boxed{1.1}$$

$$EY^P = \frac{EY^L}{S(1)} = \frac{1.1}{0.7}$$ since $S(1) = P(X > 1) = 0.7$.

$$= \Sigma\, y\, P(Y^P = y) = 1\,\frac{.3}{.7} + 2\,\frac{.4}{.7} = \boxed{1.5714}$$

17) The 100 pth percentile $\pi_p = VaR_p(X)$ = the value at risk of X at the 100p % level satisfies $F(\pi_p) = P(X \le \pi_p) = p$ where $0 < p < 1$ and X is the total loss in the year. $VaR_p(X)$ is given for several brand name RVs. Often $p \ge 0.95$.

18) The tail value at risk of X at 100p % level is $TVaR_p(X) = E(X \mid X > \pi_p) = VaR_p(X) + e_X(\pi_p)$

$$= \pi_p + \frac{E(X) - E(X \wedge \pi_p)}{1-p}$$ is given for several brand name loss RVs. Note $S_X(\pi_p) = 1-p$.

19]* The loss elimination ratio

expected loss, insurer doesn't pay

$$LER = \frac{E[X \wedge d]}{E(X)} \quad \text{if } E(X) \text{ exists.} \quad (E(X) > 0)$$

Note: $E[X \wedge d] = E(X) - E(X-d)_+ = E(X) - E(Y^L)$.

20] <u>Know</u> Given X is from a parametric distribution with parameters $\underset{\sim}{\gamma}$, be able to estimate quantities given $\hat{\underset{\sim}{\gamma}}$: plug in $\hat{\underset{\sim}{\gamma}}$ for $\underset{\sim}{\gamma}$.

21] <u>Know</u> Continuous distributions from E1 rev 1-2 (and Appendix A except for the inverse Gaussian distribution), with parameter θ are <u>scale families</u> with scale parameter θ if any other parameters $\underset{\sim}{\tau}$ are fixed, written $X \sim SF(\theta | \underset{\sim}{\tau})$. Let $a > 0$. Then $Y = aX \sim SF(a\theta | \underset{\sim}{\tau})$.

22] If $X \sim LN(\mu, \sigma)$ and $a > 0$, (4.5)

then $aX \sim LN(\mu + \ln(a), \sigma)$.

ex] Suppose $X_1, \ldots, X_n$ are iid $Geom(\beta)$

and $\hat{\beta} = \bar{X} = 1$. Estimate $P(X=0)$ and $P(X=1)$

soln] $p_k = P(X=k) = \dfrac{\beta^k}{(1+\beta)^{k+1}}$, $k = 0, 1, \ldots$

so $\hat{p}_k = \dfrac{\hat{\beta}^k}{(1+\hat{\beta})^{k+1}}$, $\hat{p}_0 = \dfrac{1}{(1+1)^1} = \boxed{\dfrac{1}{2}}$

$\hat{p}_1 = \dfrac{1}{(1+1)^{1+1}} = \boxed{\dfrac{1}{4}}$

ex) Suppose $X \sim EXP(\theta)$. If $\hat{\theta} = 100$

estimate a) $EY^P = e_X(200)$ if $d = 200$

b) $VaR_{0.5}(X) = \pi_{0.5}$

c) $E(X \wedge 200)$

soln] get these quantities from El rev PI

and plugin $\hat{\theta}$ for θ.

a) $e_X(200) = \theta \approx \hat{\theta} = \boxed{100}$

b) $VaR_{0.5}(X) = -\theta \ln(1-p) \approx -100 \ln(0.5) = \boxed{69.3147}$

c) $E(X \wedge 200) = \theta\left(1 - e^{-\frac{d}{\theta}}\right) \approx 100\left(1 - e^{-\frac{200}{100}}\right)$

$= 100\left(1 - e^{-2}\right) = \boxed{86.4665}$

ex) $X \sim EXP(\theta)$. If $\hat{\theta} = 100$, estimate $E(X)$ and $S(x)$ for $x > 0$.

Soln) $E(X) = \theta \approx \widehat{E(X)} = \hat{\theta} = \boxed{100}$

$S(x) = 1 - F(x) = e^{-x/\theta} \approx \hat{S}(x) = e^{-x/\hat{\theta}}$

$= \boxed{e^{-x/100}}$

23) Much of M404 will find estimators for parameters and show how to select brand name

distributions for X and N given data.

24) Suppose $Z \sim N(0,1)$. Then

$P(Z \le x) = \overset{\text{phi}}{\Phi}(x)$ is found from the normal table for $x > 0$

and $\Phi(x) = 1 - \Phi(-x)$ for $x < 0$.

ex) $\Phi(0.40) = \boxed{0.6554}$

	00
0.4	.6554

$\Phi(-0.50) = 1 - \Phi(0.50)$

$= 1 - .6915 = \boxed{0.3085}$

	00
0.5	.6915

$\pi_{.95} = 1.645$

	04	05
1.6	.9495	.9505

* use closest value unless there is a tie

Ch 15

1] * The method of moments estimator for $\underline{\gamma}$

Sets $E(X^j) \overset{\text{set}}{=} \frac{1}{n} \sum_{i=1}^{n} X_i^j$ for $j = 1, \ldots, k$. $(k \times 1)$

Then solve for $\gamma_1, \ldots, \gamma_k$.

2] Know The method of moments estimator for $E(X)$ is $\overline{X}$ = sample mean.