22) why are there formulas like

$$\text{"}E(\overset{w}{\text{claim}} \mid \underline{x})\text{"} = \sum E(\overset{w}{\text{claim}} \mid \underset{\theta_i}{\text{class } i})\, P(\underset{\theta_i}{\text{class } i} \mid \overset{x_1 = x}{\underline{x}})$$

and

$$\text{"}P(X_2 > c \mid \underline{x})\text{"} = \sum P(X_2 > c \mid \theta = i)\, P(\theta = i \mid \underset{x_1 = x}{\underline{x}})\ ?$$

HW 8 #14

Let $W = X_{n+1}$ ($\equiv X_{n+1} \mid \underline{x}$). The Bayesian estimate = posterior mean of $g(\theta_i)$ where $g(\theta_i) = h(w \mid \theta_i)$.

So Bayesian estimate $= E_{\pi}[g(\Theta)]$ ← posterior

$$= \sum_i g(\theta_i)\, \pi(\theta_i \mid \underline{x}) = \sum_i h(w \mid \theta_i)\, \pi(\theta_i \mid \underline{x}).$$

So $E(\text{claim} \mid \underline{x})$ has $g(\theta_i) = E(\text{claim} \mid \theta_i)$

and $P(X_2 > c \mid \underline{x})$ has $g(\theta_i) = P(X_2 > c \mid \theta_i)$.

Replace sum by integral if the posterior is a pdf.

Alternatively, $W = X_{n+1}$ ($\equiv X_{n+1} \mid \underline{x}$) has pdf

$$f(w) = f(w \mid \underline{x}) = \int f(w \mid \theta)\, \pi(\theta \mid \underline{x})\, d\theta \quad \text{by E3rev (iii)}.$$

$$\text{So } E_{W \mid \underline{x}}\, h(w) = \int h(w)\, f(w \mid \underline{x})\, dw$$

$$= \int h(w) \int f(w|\theta)\,\pi(\theta|\underline{x})\,d\theta \; dw$$

$$= \int \underbrace{\int h(w) f(w|\theta)\,dw}_{E_{w|\theta}[h(w)] = g(\theta)} \; \pi(\theta|\underline{x})\,d\theta$$

$$= \int g(\theta)\,\pi(\theta|\underline{x})\,d\theta = E_{\underset{\text{posterior}}{\pi}}[g(\Theta)]$$

Replace integrals by sums for pmfs.

23] Bayesian Credibility puts a prior on classes of risk. Let Θ_i correspond to class i. X = losses follow a different dist for each class

row	class 1	...	class k	sum
i) Prior	$\pi(\text{class } 1)$		$\pi(\text{class } k)$	1
ii) likelihood	$f(\underline{x}\mid\text{class} 1)$		$f(\underline{x}\mid\text{class } k)$	NA
iii) joint prob	product of row i) and ii) $\pi(\text{class} 1)\, f(\underline{x}\mid\text{class } 1)$		$\pi(\text{class } k)\, f(\underline{x}\mid\text{class } k)$	den. of Bayes Th
iv) posterior	ratio of row iii) term and row iii) sum $\pi(\text{class} 1\mid\underline{x}) = \frac{\pi(\text{class } 1)\, f(\underline{x}\mid\text{class} 1)}{\text{row iii) sum}}$		$\pi(\text{class} k\mid\underline{x}) = \frac{\pi(\text{class} k)\, f(\underline{x}\mid\text{class} k)}{\text{row iii) sum}}$	1
v) hypothetical or conditional mean	mean loss for class 1 $= \mu_1 = E(X\mid\text{class} 1)$		mean loss for class k $\mu_k = E(X\mid\text{class} k)$	NA
vi) Bayesian Premium contribution	$\mu_1\, \pi(\text{class} 1\mid\underline{x})$ product of row iv) and v) terms		$\mu_k\, \pi(\text{class} k\mid\underline{x})$	Bayesian premium = predicted expected value

use row iv) to find posterior prob of being in class i given data = experience = exposure

use row vi) to find Bayesian Premium = predicted expected value of next loss = "$E(X_{n+1} \mid \underline{X})$." There is enough info to get rows i) and ii). (See E3 rev 116).)

24) Often $K=2$ and there are j times as many people in class 1 as in class 2.

Then $\pi(\text{class } 1) = \frac{j}{j+1}$ and $\pi(\text{class } 2) = \frac{1}{j+1}$.

ex) portfolio of insured has good and bad drivers. There are 3 times as many good drivers as bad drivers. For good drivers, the number of claims $N \sim \text{pois}(0.1)$ and the size of claims (losses) is 1000 with prob 0.9 and 5000 with prob 0.1. For bad drivers, number of claims $N \sim \text{Pois}(0.3)$ and the size of claims is 1000 with prob 0.8 and 5000 with prob 0.2.

For an insured selected at random, she made no claims in the 1st 2 years and 1 claim for 1000 in the 3rd year. Determine the aggregate claim cost in the next year. [want $E(S) = E_{\hat{\pi}}\left(\sum_{i=1}^{N} X_i\right) = E_{\hat{\pi}}(N)\,E(X)$]

← posterior $\hat{\pi}$

	good	bad	sum
i) prior	0.75	0.25	1
ii) likelihood	0.06667	0.09758	
iii) joint prob	0.05001	0.02439	0.07440
iv) posterior	0.6721	0.3279	1
v) hypothetical mean = E(S|class i)	140	540	
vi) B. prem contr	94.0940	177.066	$\boxed{271.160}$

= Bayesian Premium

soln] $p_k = \frac{e^{-\lambda} \lambda^k}{k!}$

ii) For a good or bad driver, the likelihood of (0, 0, 1000) is

$(P_0)(P_0)(P_1 \; P(\text{claim}=1000))$. Note claim = 0 if N = 0.

For bad driver, the likelihood of (0, 0, 1000) is

$= e^{-.3}\, e^{-.3}\, \underbrace{e^{-.3}(.3)}_{P_1} .8 = 0.09758$

For good driver the likelihood is

$e^{-.1}\, e^{-.1}\, \overbrace{e^{-.1}(.1)}^{P_1} .9 = 0.06667.$

iii) $0.05001 = .75(.06667)$, $0.02439 = .25(.09758)$.

iv) $.6721 = \frac{.05001}{.07440}$, $.3279 = \frac{.02439}{.07440}$

v) For class 1, $E\left(\sum_{i=1}^{N} X_i\right) = E(N)\,E(X)$

$= 0.1\,[.9(1000) + .1(5000)] = .1(1400) = 140$

For bad, $E(N)E(X) =$

$0.3\,[.8(1000) + .2(5000)] = .3(1800) = 540$

vi) $140(.6721) = 94.094$, $.3279(540) = 177.066$

E (next aggregate claim) $= 94.094 + 177.066 = \boxed{271.160}$

see HW9 #1

25] An aggregate loss is $S = \sum_{i=1}^{N} X_i$,

and $E(S) = E(N)\,E(X)$ and $S = 0$ if $N = 0$.

26] If any distribution is continuous (likelihood), use pdfs instead of probabilities.

ex] (Same as last ex except

59.5

for good drivers, claim sizes ~ pareto ($\alpha = 2$, $\theta = 1000$) while for bad, claim sizes ~ pareto ($\alpha = 2$, $\theta = 2000$).

	good	bad	sum
i) prior	0.75	0.25	1
ii) likelihood	0.00001852	0.00003614	
iii) joint prob	0.00001389	0.000009035	0.00002293
iv) posterior	0.6058	0.3940	not 1 due to rounding
v) hyp mean	100	600	
vi) B. Prem contr	60.58	236.40	$\boxed{296.98}$

$$f(x) = \frac{\alpha \theta^{\alpha}}{(\theta + x)^{\alpha+1}}$$

soln ii) likelihood of $(0, 0, 1000) = (P_0)(P_0)(P_1 \cdot f(1000))$

good driver! $e^{-.1} \, e^{-.1} \, e^{-.1}(.1) \dfrac{2\,(1000)^2}{(1000+1000)^3} = .00001852$

bad driver: $e^{-.3} \, e^{-.3} \, e^{-.3}(.3) \dfrac{2\,(2000)^2}{(2000+1000)^3} = .00003614$

iii) eg $.75\,(.00001852) = .00001389$

iv) eg $.3940 = \dfrac{.000009035}{.00002293}$

v) $E(X) = \frac{\theta}{\alpha - 1}$

good: $E\left(\sum_{i=1}^{N} X_i\right) = E(N)\,E(X) = .1\,(1000) = 100$

bad: $E(N)\,E(X) = .3\,(2000) = 600$

vi) eg $60.58 = 100(.6058)$

So $E(\text{next aggregate claim}) = \boxed{296.98}$

Ch 20 Credibility

p555

1) Credibility theory is used to adjust premiums based on past experience (data).

2) policy holder's claims over n years

$\underbrace{(X_1, \ldots, X_n)}_{\text{experience}}$ where n is the number of years exposed to risk

eg drivers insurance, been driving for 7 years.

3) If the policyholder's experience is better than that assumed in the underlying manual

rate (pure premium), then the policyholder deserves a rate reduction.

4] Some of the policyholder's experience is due to chance and some due to the policy holder being a better or worse risk than average for the given risk class (eg 22 year old drivers with one accident in 6 years of experience).

5] The more information (exposure n) for the policy holder, the more credible the policyholder's experience, all else being equal.

§ 20.3.2

6] This section reviews the conditional pdf, marginal pdf, and conditional expectation.

7]* $E(X) = E(E(X|Y))$

$V(X) = E(V(X|Y)) + V(E(X|Y))$. $Y = \Theta$ is possible.

ex) Suppose $N|\lambda \sim \text{Poisson}(\lambda)$ where $\lambda \sim G(\alpha, \theta)$. Then $N \sim NB(\beta = \theta, r = \alpha)$.

$$E(N) = E(E(N|\lambda)) = E(\lambda) = \alpha\theta$$

$$V(N) = E(V(N|\lambda)) + V(E(N|\lambda)) =$$

$$E(\lambda) + V(\lambda) = \alpha\theta + \alpha\theta^2 = \alpha\theta(1+\theta).$$

see HW 9 #2

8) $E\left[E[h(X,Y)|Y]\right] = E[h(X,Y)]$.

9) If the X_i are independent with $E(X_i) = \mu$ and $V(X_i) = V = \sigma^2$, then $E(\bar{X}) = E\left(\frac{1}{n}\sum_{i=1}^{n} X_i\right) = \mu$ and

$$E(\hat{\sigma}_u^2) = E(\hat{V}) = E\left[\frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2\right] = V = \sigma^2.$$

§20.2 10) Classical credibility = limited fluctuation credibility theory originated in 1914. Let the policyholder experience X_j claims (number of claims $X_j = N_j$)

or losses (payment amounts) in past experience period j (often a year), 615

Or X_j is the experience from the jth policy in a group or class.

Let $E(X_j) = \xi$ and $V(X_j) = \sigma^2$.

Let M be the underlying manual rate (pure premium) based on the experience of similar Policyholders.

Assuming $X_1, \ldots, X_n$ are independent,

$E\bar{X} = \xi$ and $V(\bar{X}) = \frac{\sigma^2}{n}$.

11) Let the <u>credibility premium</u> be P_c for a time period.

$P_c = M$ ignores the policyholder's experience.

$P_c = \bar{X}$ = full credibility.

partial credibility =

$P_c = Z \overbrace{\bar{X}}^{\text{data}} + (1-Z) \overbrace{M}^{\text{other information}}$ where the <u>credibility (factor)</u> $Z \in [0,1]$ needs to