Ratemaking

1) The objectives of ratemaking are to

a) cover expected losses and expenses

b) produce rates that make adequate provision for contingencies

c) encourage loss control (want policyholder to have incentive to reduce loss costs eg smoke alarms burglar alarms)

d) satisfy state regulators,

2) Four input variables are needed to evaluate a contingent cash flow.

i) A claim frequency distribution N from recent experience data.

The average claim frequency

$$f = \frac{\text{number of claims}}{\text{units of earned exposure}}$$

ii) A loss distribution X which models the severity of the losses.

The average loss severity

$$s = \frac{\text{dollars of losses}}{\text{number of claims}} = \text{average payment per claim}$$

(iii) A rate of interest i or force of interest δ.

$e^{\delta} = 1+i$ or $\delta = \ln(1+i)$.

Typically $0 < i < 0.3$.

(v) The times at which payments are made.

3] The lost cost per unit of exposure is

loss cost = (ave. claim frequency) (ave. loss severity)

$= F(S) = \dfrac{\text{number claims}}{\text{units of earned exposure}} \quad \dfrac{\text{dollars of losses}}{\text{number of claims}}$

$= \dfrac{\text{dollars of losses}}{\text{units of earned exposure}}$

= pure premium = net premium =

$\bar{E}$[losses per unit of exposure].

ex] P66-67 Rate-making Brown

accident year	X = Year − 2003	loss cost	Y = ln(loss cost)
2003	0	119.39	4.782
2004	1	127.97	4.847
2005	2	129.89	4.867
2006	3	158.57	5.066
2007	4	188.72	5.240

Fit a (least squares) line to get #7

$$Y = a + \delta X = 4.7534 + 0.1085X$$

($Y = \hat{A}$, $\delta = \hat{\delta}$ might be better notation),

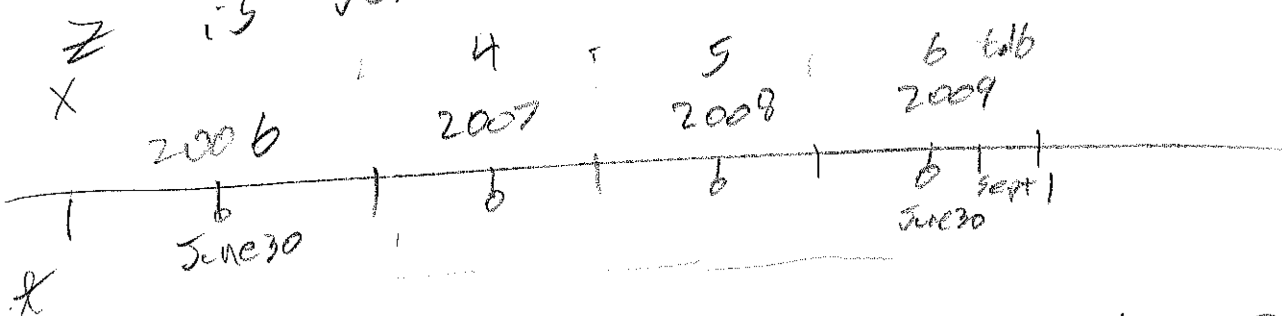
So $\delta = 0.1085 = 10.85\%$

Then Projected loss cost =

$$(\text{experience loss cost})\, e^{\delta t} \qquad (*)$$

For accident year data $t = 1 = 1$ year.

The average accident date of accident year Z is June 30, Z eg $Z = 2003$.



$t = 3 + \frac{2}{12} \approx 3.16$ Get the 2006 loss cost projected to Sept 1, 2009.

a) with the line

$$Y = 4.7534 + 0.1085(6.16) = 5.42176$$

so projected cost $= e^Y = \boxed{226.28}$

b) with (*) $158.57\left(e^{0.1085(3.16)}\right) = 158.57\, e^{0.34286}$

$$= \boxed{223.42}$$

Many other methods are possible.

4) Two methods can be used to calculate the overall average rate change.

a) Lost Cost (Pure Premium) Method

New Average Lost Cost = expected effective lost cost

$$= \frac{\text{expected dollar losses in effective period (trended and developed)}}{\text{number of earned exposure units}}$$

Then New Average Gross Rate =

$$\frac{\text{New Average Lost Cost}}{\text{Permissable Loss Ratio}}$$ where

Permissable Loss Ratio = 1 − Expense Ratio.

b) Loss Ratio Method

$$\text{Indicated Rate Change} = \frac{\text{Expected Effective Loss Ratio}}{\text{Permissable Loss Ratio}} - 1$$

where the Expected Effective Loss Ratio =

$$\frac{\text{expected dollar losses in effective period (trended and developed)}}{\text{dollars of earned premium at current rates}}$$

Then new average gross rate =
(Present average manual rate) (indicated rate change + 1)

ex) #724 Ratemaking Brown

expected effective period losses (trended and developed) 30,000,000

earned exposure units 1,000,000

earned premium at current rates 45,000,000

present average manual rate 45

permissable loss ratio = 1 - expense ratio 0.7

Find the new average gross rate.

a) Lost cost method

expected effective loss cost = $\frac{30{,}000{,}000}{1{,}000{,}000} = 30$

new average gross rate = $\frac{30}{0.7} = \boxed{42.8571}$

b) Loss Ratio Method

expected effective loss ratio = $\frac{30{,}000{,}000}{45{,}000{,}000} = \frac{2}{3} = 0.6667$

indicated rate change = $\frac{2/3}{.7} - 1 = -0.04762$

$\Rightarrow$ rate reduction of 4.762%

new average gross rate = $45\left(\frac{2/3}{0.7}\right) = \boxed{42.8571}$

ch 8 is on 403 and 404

1] support $\subseteq [0, \infty)$

2] payment per loss has 0 as a possibility when there is a loss without a payment due to a deductible.

Y^L is the per loss RV.

repeating some 403 new material!

3) For the payment per payment, the RV is not defined when there is no payment. The per payment RV is Y^P,

Note! Superscripts are sometimes left off.

§ 8.2 Deductibles

4) Insurance policies are often sold with a per loss deductible d. If loss $x \le d$, the insurance pays nothing. If the loss $x > d$, the insurance pays $x-d$.

5) The (excess loss RV $\Rightarrow$) per payment RV

$$= Y^P = (X-d) \mid X > d = \begin{cases} \text{undefined}, & X \le d \\ X-d, & X > d. \end{cases}$$

$$= Y^L \mid Y^L > 0.$$

6) The (left censored and shifted RV $\Rightarrow$) per loss RV

$$= Y^L = \begin{cases} 0, & X \le d \\ X-d, & X > d. \end{cases} = (X-d)_+ = \max(X-d, 0).$$

7) p116 180 Let X be the loss RV. For $y > 0$,

$$f_{Y^P}(y) = \frac{f_X(y+d)}{S_X(d)}, \quad S_{Y^P}(y) = \frac{S_X(y+d)}{S_X(d)},$$

← common error: omit $F_X(d)$

$$F_{Y^P}(y) = \frac{F_X(y+d) - F_X(d)}{S_X(d)} = P(X-d \le x \mid X > d) = P(X \le x+d \mid X > d) = \frac{P(d < X \le x-d)}{P(X > d)}.$$

$$h_{Y^P}(y) = \frac{f_X(y+d)}{S_X(y+d)} = h_X(y+d).$$

8) ~~p116~~ 180 Y^L is a mixture of a point mass at 0, W, say, and Y^P, so $P(W=0) = 1$.

$P(Y^L = 0) = F_X(d)$. For $y > 0$,

$$\underline{F_{Y^L}(y)} = F_X(d) \underbrace{F_W(y)}_{= \begin{cases} 1 & y \ge 0 \\ 0 & y < 0 \end{cases}} + (1 - F_X(d)) F_{Y^P}(y)$$

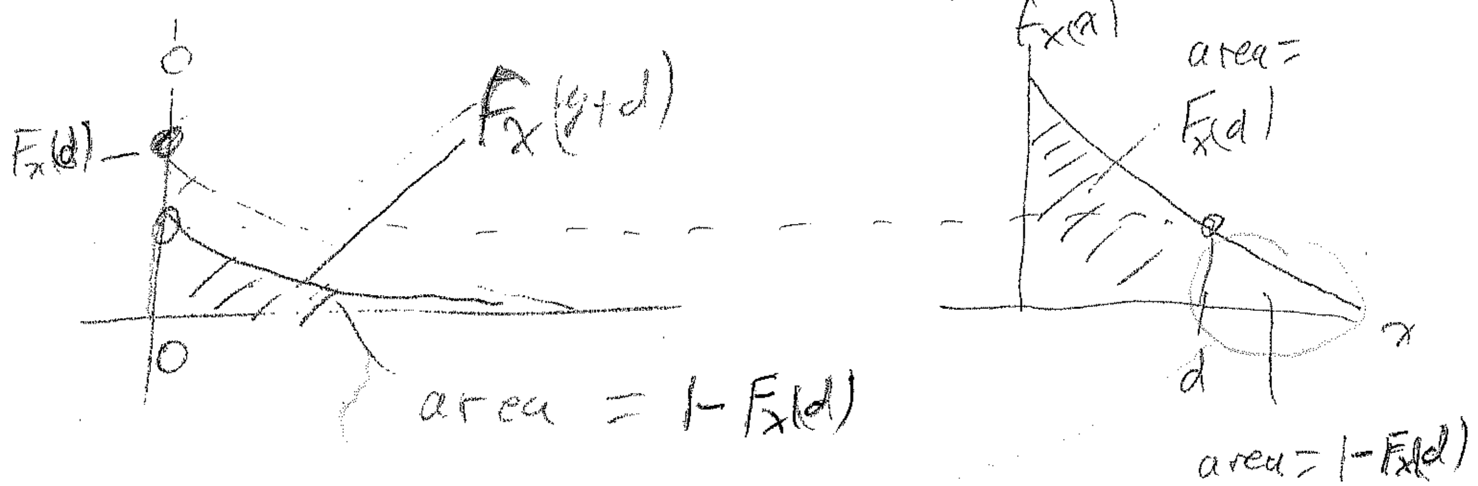
$$= F_X(d) + F_X(y+d) - F_X(d),$$

$$= \underline{F_X(y+d)},$$

(Note that: $P(X-d \le y) = P(X \le y+d) = F_X(y+d)$, $y \ge 0$.)

$$\underline{S_{Y^L}(y) = S_X(y+d)}.$$

Since Y^L is a mixture of a discrete and continuous RV, Y^L does not have a pdf. M 404 80.51



Text says pdf of Y^L for $y > 0$ is $f_X(y+d)$.

The hazard function of Y^L is not defined at 0 so is not used. Also h and H are hard to define if the RV is not continuous.

9} When counting positive claims on a per payment, changing the deductible changes the frequency with which payments are made. $P(\text{positive payment}) = S_X(d)$.

10} The per loss deductible is an ordinary deductible: the policy pays $(X-d)_+ = \max(X-d, 0) = Y^L$.

$Y^P = X$ if $X > d$ is a truncated RV

11} A franchise deductible pays $Y = \begin{cases} 0, & X \le d \\ X & X > d \end{cases}$

so it pays the full amount X if $X > d$.

12} * Assume that a deductible is an ordinary deductible unless stated otherwise.

13} If $Y^L(F)$, $Y^P(F)$ are the loss RV and per payment RV for a franchise deductible d, and $Y^L(0)$, $Y^P(0)$ those for an ordinary deductible d, then

$$E[Y^L(F)] = E(Y^L(0)) + d\,S_X(d)$$

$$E(Y^P(F)) = E(Y^P(0)) + d.$$

14} If $X \sim EXP(\theta)$, then $Y^P \sim EXP(\theta)$, $e_X(d) = \theta = E(Y^P)$.

15} If $X \sim U(0,\theta)$ and $d < \theta$, then $Y^P \sim U(0, \theta - d)$ and $e_X(d) = \frac{\theta - d}{2} = E(Y^P)$

16} If $X \sim$ (2 parameter) pareto (α, θ), then $Y^P \sim$ (2 parameter) pareto $(\alpha, \theta + d)$.

Then for $\alpha > 1$, $e_X(d) = \frac{\theta + d}{\alpha - 1}$.

17] If $X \sim$ single parameter pareto(α, θ) [M404 p81.5]

and if $d \geq \theta$, then $Y^P \sim$ (two parameter) pareto (α, d)

$$\text{If } \alpha > 1, \ e_X(d) = \begin{cases} \dfrac{d}{\alpha - 1} & d \geq \theta \\[2ex] \dfrac{\alpha(\theta - d) + d}{\alpha - 1} & d \leq \theta \end{cases}$$

18] * the mean excess loss

$$\underline{E(Y^P) = e_X(d)} = \frac{E(X) - E(X \wedge d)}{S_X(d)} = \frac{E[(X-d)_+]}{S_X(d)} = \frac{E(Y^L)}{S_X(d)}$$

$$= \frac{\int_d^\infty (x-d) f_X(x)\,dx}{S_X(d)} = \frac{\int_d^\infty S_X(x)\,dx}{S_X(d)}.$$

$$E[X] = E(X \wedge d) + e_X(d) S_X(d).$$

19] Tables give $VaR_p(X)$ (percentile) and

$$TVaR_p(X) = VaR_p(X) + e_X(\pi_p) = \pi_p + e_X(\pi_p)$$

20] For the franchise deductible,

$$f_{Y^P}(y) = \frac{f_X(y)}{S_X(d)}, \quad y > d \qquad \text{(truncated RV)}$$