$$S_{Y^P}(y) = \begin{cases} 1 & 0 \le y \le d \\ \dfrac{S_X(y)}{S_X(d)} & y > d \end{cases}$$

$$F_{Y^P}(y) = \begin{cases} 0 & 0 \le y \le d \\ \dfrac{F_X(y) - F_X(d)}{S_X(d)}, & y > d \end{cases}$$

$$h_{Y^P}(y) = \begin{cases} 0 & 0 < y < d \\ h_X(y) & y > d \end{cases}$$

For Y^L ← mixture of discrete and contin

the pdf is not defined but text

says $f_{Y^L}(y) = \begin{cases} F_X(d) & y = 0 \\ f_X(y) & y > d \end{cases}$

$$S_{Y^L}(y) = \begin{cases} S_X(d) & 0 \le y \le d \\ S_X(y) & y > d \end{cases}$$

$$F_{Y^L}(y) = \begin{cases} F_X(d) & 0 \le y \le d \\ F_X(y) & y > d \end{cases}$$

$$h_{Y^L}(y) = \begin{cases} 0 & 0 < y < d \\ h_X(y) & y > d \end{cases}$$

21] For a franchise deductible

$E(Y^L)$ = expected cost per loss = $E[Y^L(F)]$

$$E(X) - E(X \wedge d) + d\, S_X(d)$$

$E(Y^P)$ = expected cost per payment = $\dfrac{E(Y^L)}{S_X(d)}$

$= E[Y^P(F)]$.

If $Y^P(O)$ and $Y^P(F)$ denote the RVs with ordinary and franchise deductibles

then $E[Y^P(F)] = E[Y^P(O)] + d$:

when there is a payment, that for a policy with a franchise deductible exceeds that for a policy with an ordinary deductible by d. See 13].

§8.3 22]* The loss elimination ratio (LER) is the ratio of the decrease in the expected payment with an ordinary deductible to the expected payment without the deductible.

$$LER = \frac{E[X \wedge d]}{E[X]} \text{ if } E(X) \text{ exists.}$$

Note that $\dfrac{E[X]-E[Y^L]}{E(X)} = \dfrac{E[X]-[E(X)-E(X\wedge d)]}{E(X)}$

$$= \frac{E[X \wedge d]}{E(X)} = LER.$$

23) Interpretation: if $LER = 0.36$ then 36% of losses can be eliminated by using deductible d.

24) Let the annual inflation rate be r. Sometimes you are told there is uniform inflation $1+r$. (Except for catastrophic inflation rates expect $0<r<1$.) After inflation the new loss RV

$Y = (1+r)X$.

25} Let X be from a scale family with scale parameter θ when the other parameters $\underset{\sim}{I}$ are fixed. $X \sim SF(\theta | \underset{\sim}{I})$. Then

$$Y = (1+r)X \sim SF((1+r)\theta | \underset{\sim}{I}).$$

26} Nearly all the continuous distributions in App A with parameter θ are "scale families" with scale parameter θ. The inverse Gaussian is an exception.

If $X \sim LN(\mu, \sigma)$, then

$$Y = (1+r)X \sim LN(\mu + \ln(1+r), \sigma).$$

27} $X \sim SF(\theta | \underset{\sim}{I})$ for $\underset{\sim}{I}$ fixed if the cdf $F_X(x) = G(\frac{x}{\theta})$ depends on x and θ only through $\frac{x}{\theta}$.

i) $F_X(x) = G(\frac{\theta}{x}) = G\left(\frac{1}{x/\theta}\right)$.

ii) $F_X(x) = G\left(\frac{x}{x+\theta}\right) = G\left(\frac{x/\theta}{x/\theta + 1}\right)$

iii) $F(x) = G\left(\frac{\theta}{x+\theta}\right) = G\left(\frac{1}{\frac{x}{\theta}+1}\right)$.

★2-8) Recognizing dist's

Support $x>0$ unless given

dist	pdf	
Uniform	c ,	$d \le x \le u$
beta	$c\,x^{a-1}(\theta-x)^{b-1}$	$0<x<\theta$, M483 beta has $\theta=1$
Exponential	$c\,e^{-x/\theta}$	
weibull	$c\,x^{\tau-1}\,e^{-x^{\tau}/\theta^{\tau}}$	
Gamma	$c\,x^{a-1}\,e^{-x/\theta}$	
(two parameter) pareto	$\frac{c}{(x+\theta)^{a+1}}$	
single parameter pareto	$\frac{c}{x^{a+1}}$,	$x \ge \theta$
LN	$\frac{c}{x}\exp\left[\frac{-(\ln(x)-\mu)^2}{2\sigma^2}\right]$	

ex) If $X \sim$ two parameter pareto $(\theta=6000, \alpha=4)$

then $Z=(1.1)X \sim$ two parameter pareto $(\theta=\underbrace{6600}_{1.1(6000)}, \alpha=4)$.

29]* If $Y = (1+r)X$ then M407 845

$$E[Y \wedge d] = (1+r)\, E\left[X \wedge \frac{d}{1+r}\right]$$

$$E(Y) = (1+r)\, E(X)$$

$$F_Y(d) = P(Y \le d) = P\big((1+r)X \le d\big) = F_X\left(\frac{d}{1+r}\right)$$

$$S_Y(d) = 1 - F_Y(d) = S_X\left(\frac{d}{1+r}\right).$$ Watch out for $S_Y(d) = 0$.

30]* If $Y = (1+r)X$ and $X \sim SF(\theta \mid \tilde{I})$, then $Y \sim SF((1+r)\theta \mid \tilde{I})$. Using formulas for the modified dist for Y may be easier than using formulas from 29].

31]* Unt. App. A for single parameter pareto (α, θ). gives formula for $E[X \wedge d]$ that is valid if $d > \theta$. If $d \le \theta$, $E[X \wedge d] = d$ since $P(X \ge d) = P(X \ge \theta) = 1$.

E1 rev :0) has $E[X \wedge x]$ for $x \le \theta$ and $x > \theta$.

32] For an ordinary deductible of d after uniform inflation of $1+r$

$$E(Y^L) = (1+r)\left[E(X) - E\left(X \wedge \frac{d}{1+r}\right)\right]$$

and $E(Y^P) = \dfrac{E(Y^L)}{S_X\left(\frac{d}{1+r}\right)}$ if $S_X\left(\frac{d}{1+r}\right) \neq 0$.

ex3 $d = 500$, $X \sim$ pareto$(\alpha = 3, \theta = 2000)$

$r = 10\%$.

Before inflation,

$E(Y^L) = E(X) - E(X \wedge d) =$

$\dfrac{\theta}{\alpha-1} - \dfrac{\theta}{\alpha-1}\left[1 - \left(\dfrac{\theta}{d+\theta}\right)^{\alpha-1}\right] =$

$\dfrac{2000}{2}\left(\dfrac{2000}{500+2000}\right)^2 = 1000\left(\frac{20}{25}\right)^2 = \boxed{640}$

$F(d) = 1 - \left(\dfrac{\theta}{d+\theta}\right)^\alpha$ so $S(d) = \left(\frac{20}{25}\right)^3 = 0.512$

so $E(Y^P) = e_X(d) = \dfrac{\theta+d}{\alpha-1} = \dfrac{2500}{2} = \dfrac{640}{0.512} = \boxed{1250}$

After inflation method i) use $X_{new} \sim$ pareto$(\alpha = 3, \theta = 2200)$

$= 1.1(2000)$

$EY^L = \dfrac{\theta}{\alpha-1}\left(\dfrac{\theta}{d+\theta}\right)^{\alpha-1} = \dfrac{2200}{2}\left(\dfrac{2200}{500+2200}\right)^2 = \boxed{730.3155}$

$EY^P = \dfrac{\theta+d}{\alpha-1} = \dfrac{2700}{2} = \boxed{1350}$

method (ii) $E(Y^L) = (1+r)\left[E(X) - E\left(X \wedge \frac{d}{1+r}\right)\right]$

$$= 1.1 \ \frac{\theta}{\alpha-1}\left(\frac{\theta}{\frac{d}{1+r}+\theta}\right)^{\alpha-1} = 1.1 \ \frac{2000}{2}\left(\frac{2000}{\frac{500}{1.1}+2000}\right)^{2}$$

$$= \boxed{730.3155}$$

$$S_X\left(\frac{d}{1+r}\right) = 1 - F_X\left(\frac{d}{1+r}\right) = \left(\frac{\theta}{\frac{d}{1+r}+\theta}\right)^{\alpha}$$

$$= \left(\frac{2000}{\frac{500}{1.1}+2000}\right)^{3} = \underline{0.5410}$$

$$E(Y^P) = \frac{730.3155}{0.5410} = \boxed{1350.00}$$

<u>Warning</u> it is rather easy to make mistakes with both methods.

For method 1 $\quad \theta_{new} = (1+r)\theta \qquad$ dont multiply α by $1+r$.

33} * $E(Y^L) = E[(X-d)_+] = E(X) - E[X \wedge d]$

So $E[X \wedge d] = E(X) - E[(X-d)_+] = E(X) - E(Y^L)$.

$LER = \dfrac{E[X \wedge d]}{E(X)}$.

§8.4 34} For a policy limit, the limited loss RV $W = X \wedge U = \min(X, U)$

$$= \begin{cases} X, & x < U \\ U, & x \geq U, \end{cases}$$ a right censored RV.

$E[X \wedge U]$ = limited expected value.

$$F_W(y) = F_{X \wedge U}(y) = \begin{cases} F_X(y) & y < U \\ 1 & y \geq U \end{cases}$$

$$f_W(y) = F''_{X \wedge U}(y) = \begin{cases} f_X(x) & y < U \\ 1 - F_X(U) & y \geq U \end{cases}$$

actually a mixture of a discrete and cont'n RV

35} Let $Y = (1+r)X = X_{new}$

method 1 $E(Y \wedge U) = E[X_{new} \wedge U] = (1+r)\, E\left[X \wedge \frac{U}{1+r}\right]$

method 2 If $X \sim SF(\theta | \mathcal{I})$ then $X_{new} \sim SF((1+r)\theta / \mathcal{I})$

get $E[X_{new} \wedge U]$ using the table formulas for the modified distr. M404 §6.5

§8.5 36} For a coinsurance policy, the insurance company pays αX of the loss for some $\alpha \in (0, 1]$.

For coinsurance with a deductible, the insurance company pays $\alpha (X-d)_+$.

37}* For insurance with a deductible d and a policy limit = max payment = $U-d$, the "maximum covered loss" = U = $U-d+d$ (Loss > U is not fully covered), then

$$Y^L = \begin{cases} 0, & X < d \\ X-d, & d \le X < U \\ U-d, & X \ge U \end{cases} = X \wedge U - X \wedge d$$

$Y^P = Y^L \mid X > d$.

38}* Consider an insurance with an ordinary deductible d, limit $(U-d)$, coinsurance α, and inflation r. The per loss RV is

$$Y^L = \begin{cases} 0, & X < \frac{d}{1+r} \\ \alpha[(1+r)X-d], & \frac{d}{1+r} \le X < \frac{U}{1+r} \\ \alpha(U-d), & X \ge \frac{U}{1+r} \end{cases}$$