Math 473 Exam 1 Spring 2021 Name____________________

YOU ARE BEING GRADED FOR WORK

1) Suppose

$$S(t) = 1 - \left(\frac{t}{\omega}\right)^{\theta}$$

for $0 < t < \omega$ where $\theta > 0$. Find the following quantities for $0 < t < \omega$.

a) $F(t)$

b) $f(t)$

c) $h(t)$

d) $H(t)$

2) The Smith (2002, pp. 83-84) data is for survival times for 913 men and women with malignant melanoma treated between 1944 and 1960. Use the life table estimator to compute the estimated survival function $\hat{S}_L(t_i)$ by filling in the table below. Show what you multiply to find $\hat{S}_L(t_i)$. Then plot the function.

I_j	d_j	c_j	n_j	n'_j	$\frac{n'_j - d_j}{n'_j}$	$\hat{S}_L(t_i)$
$[0, 1)$	312	96	913	865.0	0.6393	1.0000
$[1, 2)$	96	74	505	468.0	0.7949	0.6393
$[2, 3)$	45	62	335	304.0	0.8520	0.5082
$[3, 4)$	29	30	228	213.0	0.8638	0.4330
$[4, 5)$	7	40	169	149.0	0.9530	0.3740
$[5, 6)$	9	37	122	103.5	0.9130	0.3564
$[6, 7)$	3	17	76	67.5	0.9556	0.3254
$[7, 8)$	1	12	56			
$[8, 9)$	3	8	43			
$[9, \infty)$	32	0	32			

```
1 _
  |
  |
  |
  |
  |
  |
  |
  |
  |
  |
  |------------------------------------------------------------------
  0      1      2      3      4      5      6      7      8      9      10
```

3) Survival times for 11 patients with AM leukemia are given below. The data is from Smith (2002, p. 115) and Miller (1981, p. 49).
9, 13, 13+, 18, 23, 28+, 31, 34, 45+, 48, 161+
Compute the Kaplan Meier survival function $\hat{S}_K(t_i)$ by filling in the table below. Show what you multiply to find $\hat{S}_k(t_i)$. Then plot the function.

$t_{(j)}$	γ_j	t_i	n_i	d_i	$\hat{S}_K(t_i)$
		$t_0 = 0$			$\hat{S}_K(0) = 1$
9	1	9	11	1	$\hat{S}_K(9) =$
13	1	13			$\hat{S}_K(13) =$
13	0				
18	1	18			$\hat{S}_K(18) =$
23	1	23			$\hat{S}_K(23) =$
28	0				
31	1	31			$\hat{S}_K(31) =$
34	1	34			$\hat{S}_K(34) =$
45	0				
48	1	48			$\hat{S}_K(48) =$
161	0				

```
1 _
  |
  |
  |
  |
  |
  -----|-----|-----|-----|-----|-----|-----|-----|-----|-----|---
  0    16    32    48    64    80    96   112   128   144   160
```

4) Survival times in months for eleven male patients treated with pulmonary metastasis are given below. Data is from Collett (2003, p. 16). after being inoculated
11, 13, 13, 13, 13, 13, 14, 14, 15, 15, 17
Compute the empirical survival function $\hat{S}_E(t_i)$ by filling in the table below.

$t_{(j)}$	t_i	d_i	$\hat{S}_E(t_i)$
	$t_0 = 0$		$\hat{S}_E(0) = 1 = \frac{7}{7}$
11	11	1	$\hat{S}_E(11) =$
13			
13			
13			
13			
13			$\hat{S}_E(13) =$
14			
14			$\hat{S}_E(14) =$
15			
15			$\hat{S}_E(15) =$
17			$\hat{S}_E(17) =$

5) The following 20 survival times are listed from smallest to largest.

```
  15   22   38   49   62   71   91  102   131  145
177  198  247  279  319  359  469  526  703  790
```

Find the empirical survival function value at 247:

$$\hat{S}_E(247) = \frac{\# > 247}{n}$$

where $\# > 247$ is the number of survival times > 247.

time	n.risk	n.event	survival	std.err	lower 95% CI	upper 95% CI
9	11	1	0.909	0.0867	0.7392	1.000
13	10	1	0.818	0.1163	0.5903	1.000
18	8	1	0.716	0.1397	0.4422	0.990
23	7	1	0.614	0.1526	0.3145	0.913
31	5	1	0.491	0.1642	0.1691	0.813
34	4	1	0.368	0.1627	0.0494	0.687
48	2	1	0.184	0.1535	0.0000	0.485

6) The length of times of remission (time until relapse) in acute myelogeneous leukemia under maintenance chemotherapy for 11 patients is
$9, 13, 13+, 18, 23, 28+, 31, 34, 45+, 48, 161+$. From the output above what is the 95% CI for $S_Y(34)$?

7) Find the 95% classical CI for $S_Y(32)$ if $n = 9$ and $\hat{S}_E(32) = 6/9$.

8) Find the 95% plus four CI for $S_Y(32)$ if $n = 9$ and $\hat{S}_E(32) = 6/9$.

9) Let $\mu = E(Y)$ be the mean survival time. SAS output states that for the myelomatomis data, $\hat{\mu} = 562.76$ and $\text{SE}(\hat{\mu}) = 117.32$. Find a 95% CI for μ.

10) A survival function for treatment A is plotted below.

```
1.0|_______
.9 |       |
.8 |       |
.7 |       |_____________
.6 |                     |_____
.5 |                           |
.4 |                           |_______
.3 |                                   |
.2 |                                   |
.1 |                                   |___________________
0  |_______________________________________________________|___
       |   |   |   |   |   |   |   |   |   |   |   |   |   |
       10  20  30  40  50  60  70  80  90  100 110 120 130 140
```

a) Estimate when 50% of the patients from treatment A have died. Show the over and down lines.

b) Suppose treatment B had higher hazard than treatment A for $0 < t < 140$. Would you expect the survivor function for treatment B to be lower or higher that that for treatment A in the above plot?