Math 473 Exam 1 Spring 2017 Name____________________

YOU ARE BEING GRADED FOR WORK

1) Suppose

$$F(t) = 1 - \exp\left[\frac{-(e^t - 1)}{\lambda}\right]$$

TEV

for $t > 0$ where $\lambda > 0$ and $t > 0$. Find the following quantities for $t > 0$.

a) $S(t)$ $= 1 - F(t) = \boxed{\exp\left(\frac{-(e^t - 1)}{\lambda}\right)} = e^{\frac{-e^t + 1}{\lambda}}$

b) $f(t)$ $= F'(t) = -S'(t) = h(t)S(t)$

diff wrong −3

$= -\exp\left(\frac{-(e^t - 1)}{\lambda}\right)\left(-\frac{e^t}{\lambda}\right) = \boxed{\frac{e^t}{\lambda}\exp\left(\frac{-(e^t - 1)}{\lambda}\right)} = \frac{e^t}{\lambda} e^{\frac{-e^t + 1}{\lambda}}$

$= \frac{e^{\left[t - \frac{(e^t + 1)}{\lambda}\right]}}{\lambda}$

c) $h(t)$ $= \frac{d}{dt}H(t) = \frac{f(t)}{S(t)} = \boxed{\frac{1}{\lambda}e^t}$

d) $H(t)$ $= -\log(S(t)) = \boxed{\frac{e^t - 1}{\lambda}}$

one of a)-d) negative −3

$= \int_0^t h(u)\,du = \int_0^t \frac{1}{\lambda} e^u\,du = \frac{1}{\lambda} e^u \Big|_0^t = \frac{e^t - 1}{\lambda}$

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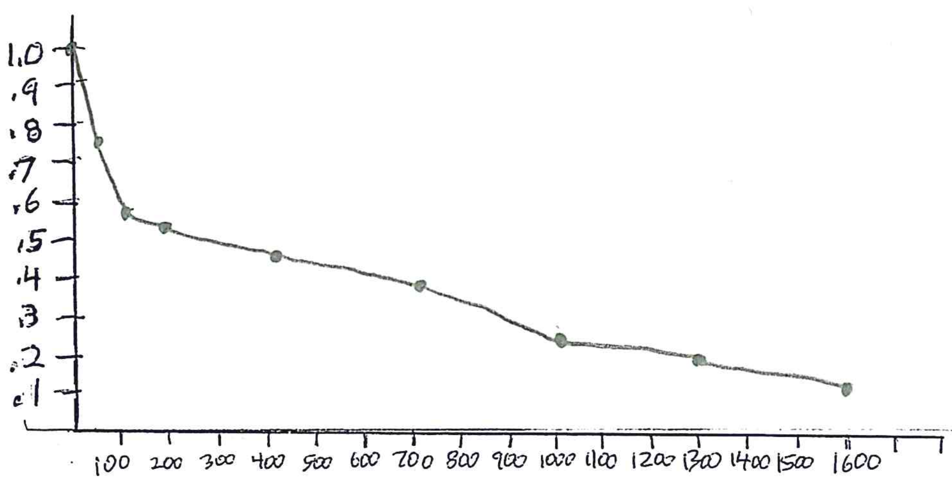
2) The Allison (1996, p. 49) data is for survival times in days for 68 heart transplant patients. Use the life table estimator to compute the estimated survival function $\hat{S}_L(t_i)$ by filling in the table below. Show what you multiply to find $\hat{S}_L(t_i)$. Then plot the function.

I_j	d_j	c_j	n_j	$n'_j = n_j - \frac{c_j}{2}$	$\frac{n'_j - d_j}{n'_j}$	$\hat{S}_L(t_i)$	$= \hat{S}_L(t_{i-1}) \frac{n_i' - d_i}{n_i}$
[0, 50)	16	3	68	66.5	0.7594	1.0000	
[50, 100)	11	0	49	49	0.7755	0.7594	$= \hat{S}(50)$
[100, 200)	4	2	38	37	0.8919	0.5889	$= \hat{S}(100)$
[200, 400)	5	4	32	30	0.8333	0.5253	$= \hat{S}(200)$
[400, 700)	2	6	23	20	0.9000	0.4377	$= \hat{S}(400)$
[700, 1000)	4	3	15	13.5	0.7037	0.3939	
[1000, 1300)	1	2	8	7	.8571	0.2772	= 0.3939(.7037)
[1300, 1600)	1	3	5	3.5	0.7143	0.2376	= 0.2772(.8571)
[1600, ∞)	0	1	1	0.5	1.0	0.1697	= 0.2376(.7143)



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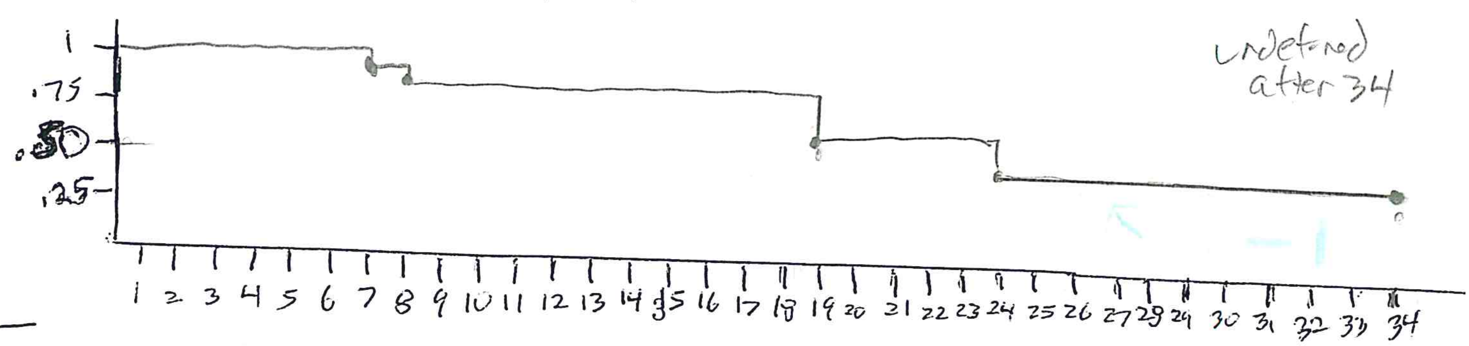
Collett p50

3) Survival times for 15 patients with melanoma are given below.
7, 8, 8, 8+, 12+, 16+, 17+, 17+, 18+, 19, 21+, 24, 24+, 27+, 34+
Compute the Kaplan Meier survival function $\hat{S}_K(t_i)$ by filling in the table below. Show what you multiply to find $\hat{S}_k(t_i)$. Then plot the function.

$t_{(j)}$	γ_j	t_i	n_i	d_i	$\hat{S}_K(t_i)$
		$t_0 = 0$			$\hat{S}_K(0) = 1$
7	1	7	15	1	$\hat{S}_K(7) = 1\left(1-\frac{1}{15}\right) = \frac{14}{15} = 0.9333$
8	1	8	14	2	$\hat{S}_K(8) = \frac{14}{15}\left(1-\frac{2}{14}\right) = 0.8000 \approx .79997$
8	1				
8	0				
12	0				
16	0				
17	0				
17	0				
18	0				
19	1	19	6	1	$\hat{S}_K(19) = .8\left(1-\frac{1}{6}\right) = .6666 = \frac{2}{3}$
21	0				
24	1	24	4	1	$\hat{S}_K(24) = \frac{2}{3}\left(1-\frac{1}{4}\right) = .5000$
24	0				
27	0				
34	0				

roughly 7 if 1st 3 are right



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4) Survival times for 15 patients with melanoma are given below.
7, 8, 8, 8, 12, 16, 17, 17, 18, 19, 21, 24, 24, 27, 34
Compute the empirical survival function $\hat{S}_E(t_i)$ by filling in the table below.

$t_{(j)}$	t_i	d_i	$\hat{S}_E(t_i)$	
	$t_0 = 0$		$\hat{S}_E(0) = 1 = \frac{15}{15}$	
7	7	1	$\hat{S}_E(7) = (15-1)/15 = 14/15$	
8				
8				
8	8	3	$\hat{S}_E(8) = (14-3)/15 = 11/15$	
12	12	1	$\hat{S}_E(12) = (11-1)/15 = 10/15$	
16	16	1	$\hat{S}_E(66) = (10-1)/15 = 9/15$	
17				
17	17	2	$\hat{S}_E(17) = (9-2)/15 = 7/15$	
18	18	1	$\hat{S}_E(18) = (7-1)/15 = 6/15$	
19	19	1	$\hat{S}_E(19) = (6-1)/15 = 5/15$	$= .3333$
21	21	1	$\hat{S}_E(21) = (5-1)/15 = 4/15$	$= .2667$
24				
24	24	2	$\hat{S}_E(24) = (4-2)/15 = 2/15$	$= .1333$
27	27	1	$\hat{S}_E(27) = (2-1)/15 = 1/15$	$= .06667$
34	34	1	$\hat{S}_E(34) = (1-1)/15 = 0$	

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5) The following 20 survival times are listed from smallest to largest.

15	22	38	49	62	71	91	102	131	145
177	198	247	279	319	359	469	526	703	790

Find the empirical survival function value at 320:

$$\hat{S}_E(320) = \frac{\# > 320}{n}$$

where $\# > 320$ is the number of survival times > 320.

$$= \boxed{\frac{5}{20} = \frac{1}{4} = 0.25}$$

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Q1d21

e

6) The survival functions plotted below are for steroid induce remission times (weeks) for leukemia patients. One group of 21 patients was given the drug 6-MP, and a second group was given a placebo. (Want remission times to be long, like you want survival times to be long for sick people.)

4 a) For the patients who received a placebo, estimate when 50% of these have died. Show the over and down lines.

$\boxed{2.1}$ (2-2.2) units are not weeks

3 b) Which treatment (6-MP or placebo) seems to be better? Explain briefly.

$\boxed{\text{6MP, its survival curve is higher}}$

3 c) If the hazard functions were plotted, which treatment would have the higher hazard (6-MP or placebo)?

$\boxed{\text{placebo}}$

(lower survival means higher hazard)

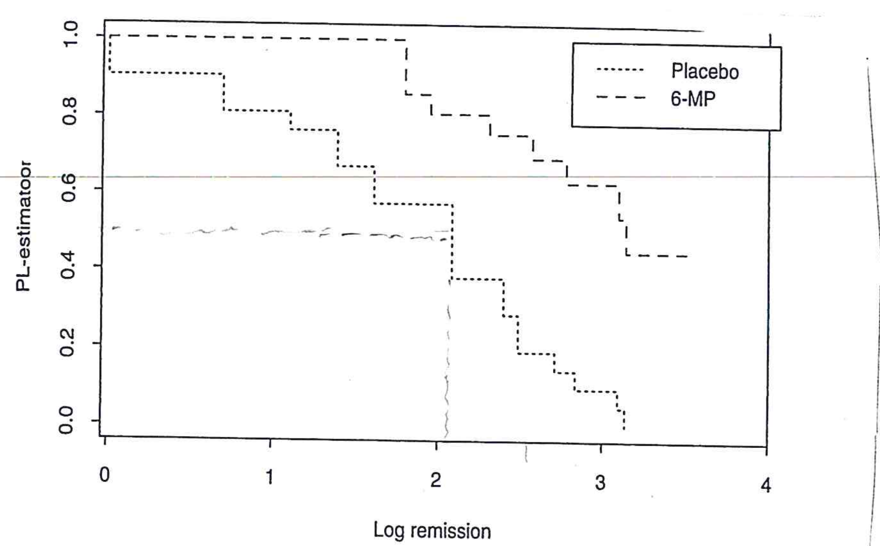

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Figure 6.3 *The PL-estimator for the 6-MP and Placebo groups when survival is measured on the* $\log_e$ *scale.*

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time	n.risk	n.event	survival	std.err	lower 95% CI	upper 95% CI
9	11	1	0.909	0.0867	0.7392	1.000
13	10	1	0.818	0.1163	0.5903	1.000
18	8	1	0.716	0.1397	0.4422	0.990 ←
23	7	1	0.614	0.1526	0.3145	0.913
31	5	1	0.491	0.1642	0.1691	0.813
34	4	1	0.368	0.1627	0.0494	0.687
48	2	1	0.184	0.1535	0.0000	0.485

7) The length of times of remission (time until relapse) in acute myelogeneous leukemia under maintenance chemotherapy for 11 patients is 9, 13, 13+, 18, 23, 28+, 31, 34, 45+, 48, 161+. From the output above what is the 95% CI for $S_Y(18)$?

$(0.4422, 0.990)$

← some students skipped this problem

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8) Find the 95% classical CI for $S_Y(13)$ if $n = 11$ and $\hat{S}_E(13) = 5/11$.

$\hat{S}(13) \pm 1.96\sqrt{\frac{\hat{S}(13)(1-\hat{S}(13))}{n}} = \frac{5}{11} \pm 1.96\sqrt{\frac{\frac{5}{11}\frac{6}{11}}{11}}$

$= .4545 \pm 1.96(.1501) = .4545 \pm .2943$

$= \boxed{(0.1602, 0.7488)}$

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9) Find the 95% plus four CI for $S_Y(13)$ if $n = 11$ and $\hat{S}_E(13) = 5/11$.

$\tilde{P}_{13} = \frac{n\hat{S}(13)+2}{n+4} = \frac{11\left(\frac{5}{11}\right)+2}{11+4} = \frac{7}{15}$ $\quad CI = \tilde{P} \pm 1.96\sqrt{\frac{\tilde{P}(1-\tilde{P})}{n+4}}$

$= \frac{7}{15} \pm 1.96\sqrt{\frac{\frac{7}{15}\frac{8}{15}}{15}} = .4667 \pm .2525 = \boxed{(.2142, .7192)}$

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10) SAS output for the myelomatomis data says that $\hat{S}_K(8) = 0.92$ and $\text{SE}(\hat{S}_K(8)) = 0.0543$. Find a 95% CI for $S(8)$.

$\hat{S}_K(8) \pm 1.96\ SE[\hat{S}_K(8)]$

wrong formula -5

$= .92 \pm 1.96(.0543) =$

round down to 1

$.92 \pm .1064 = (.8136, 1)$

or

$(= (.8136, 1.0264))$

or

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