Calculus review (Math 150):

Limits. Let $\lim_{x \to c^*} f(x)$ denote $\lim_{x \to c} f(x)$, $\lim_{x \to c^-} f(x)$, or $\lim_{x \to c^+} f(x)$.
Know the definition of limit, right and left limits, continuity, and infinite limits.
$\lim_{x \to c^*} f(x) = f(c)$ if f is continuous at c.
$\lim_{x \to c^*} f(g(x)) = f(g(c))$ if g is continuous at c and f is continuous at $g(c)$.

Derivatives. The *product rule* is (fg)' = f'(x)g(x) + f(x)g'(x).
The *quotient rule* is $(\frac{n(x)}{d(x)})' = \frac{dn' - nd'}{d^2}$.
Know how to find 2nd, 3rd, etc derivatives.
The *chain rule* is [f(g(x))]' = [f'(g(x))][g'(x)].
Know the derivative of ln x and e^x and know the chain rule with these functions.

Integrals. The *indefinite integral* $\int f(x)dx = F(x) + C$ where the *antiderivative* $F(x)$ satisfies $F'(x) = f(x)$. Alternatively, $\int f'(x)dx = f(x) + C$. The *power rule* (the integrand $f(x) = x^n, n \neq -1$) is very important. So are the trig rules, the exponential rules, and the rule for $f(x) = 1/x$.

Know the *(1st) Fundamental theorem of calculus.* $\int_a^b f(x)dx = F(b) - F(a)$ where the antiderivative F satisfies $F'(x) = f(x)$. Alternatively, $\int_a^b f'(x)dx = f(b) - f(a)$.

The 2nd Fundamental theorem of Calculus is $\frac{d}{dx}\int_a^x f(t)dt = f(x)$.

Know that $\frac{d}{dx}\int_a^{g(x)} f(t)dt = f(g(x))g'(x)$.

Know how to find *definite integrals* with the (1st) Fundamental theorem of calculus. $\int_a^b f(x)dx = F(b) - F(a)$ where the antiderivative F satisfies $F'(x) = f(x)$. Alternatively, $\int_a^b f'(x)dx = f(b) - f(a)$.

Indefinite integrals by u-substitution. $I = \int f(g(x))g'(x)dx = F(g(x)) + C$ where $F'(x) = f(x)$. The integrand $f(g(x))g'(x)$ consists of the product of a composite function (a function with an outer function f and an inner function g) with g'(x), the derivative of the inner function. Let $u = g(x)$. Then $du = g'(x)dx$ and $I = F(u) + C = F(g(x)) + C$. Note that F is the antiderivative of the outer function f.

Definite integrals by u-substitution. $I = \int_a^b f(g(x))g'(x)dx = \int_{g(a)}^{g(b)} f(u)du = \int_c^d f(u)du = F(u)|_c^d = F(d) - F(c) = F(u)|_{g(a)}^{g(b)} = F(g(x))|_a^b = F(g(b)) - F(g(a))$ where $F'(x) = f(x)$, $u = g(x)$, $du = g'(x)dx$, $d = g(b)$, and $c = g(a)$.

Tricks for u-substitution: the integrand has to have the form $h(x)\ f(g(x))$. Then $u = g(x)$. To make $h(x)dx = g'(x)dx$, sometimes the integral has to be multiplied by one in the form c/c where c is some constant. In particular,
a) If $g(x) = ax + b$, then $h(x) \equiv 1$ (or any other constant).
b) If the integrand is $h(x)/g(x)$, try $u = g(x)$.
c) If the integrand is $h(x)/(g(x))^k$, try $u = g(x)$.
d) If the integrand is $h(x)e^{g(x)}$, try $u = g(x)$.

Know the relationship between the integral and area under the graph.

MIN-MAX. Know how to find the max and min of a function h that is continuous on an interval [a,b] and differentiable on (a, b). Solve $h'(x) \equiv 0$ and find the places where h'(x) does not exist. These values are the **critical points**. Evaluate h at a, b, and the critical points. One of these values will be the min and one the max.

Assume h is continuous. Then a critical point θ_o is a local max of $h(\theta)$ if h is increasing for $\theta < \theta_o$ in a neighborhood of θ_o and if h is decreasing for $\theta > \theta_o$ in a neighborhood of θ_o (and θ_o is a global max if you can remove the phrase "in a neighborhood of θ_o"). The first derivative test is often used.

If h is strictly concave ($\frac{d^2}{d\theta^2}h(\theta) < 0$ for all θ), then any local max of h is a global max.

Suppose $h'(\theta_o) = 0$. The 2nd derivative test states that if $\frac{d^2}{d\theta^2}h(\theta_o) < 0$, then θ_o is a local max. Know when the test fails (eg if h"$(\theta_o) = 0$ at a critical number of f).

Know. If $h(\theta)$ is a continuous function on an interval with endpoints $a < b$ (not necessarily finite), and differentiable on (a, b) and if the **critical point is unique**, then the critical point is a **global maximum** if it is a local maximum (because otherwise there would be a local minimum and the critical point would not be unique).

Know the definitions of *increasing, decreasing, concave up and down, inflection points.*
Math 250: Know integration by parts.

integration by parts → 1. $\int u\,dv = uv - \int v\,du$

power rule → 2. $\int u^n\,du = \frac{1}{n+1}u^{n+1} + C,\ n \neq -1$

3. $\int \frac{1}{u}\,du = \ln|u| + C$

exp function → 4. $\int e^u\,du = e^u + C$

5. $\int a^u\,du = \frac{a^u}{\ln a} + C$

6. $\int \sin u\,du = -\cos u + C$

7. $\int \cos u\,du = \sin u + C$

usually not important for M483

8. $\int \sec^2 u\,du = \tan u + C$

9. $\int \csc^2 u\,du = -\cot u + C$

10. $\int \sec u \tan u\,du = \sec u + C$

11. $\int \csc u \cot u\,du = -\csc u + C$

12. $\int \tan u\,du = \ln|\sec u| + C$

13. $\int \cot u\,du = \ln|\sin u| + C$

14. $\int \sec u\,du = \ln|\sec u + \tan u| + C$

15. $\int \csc u\,du = \ln|\csc u - \cot u| + C$

16. $\int \frac{1}{\sqrt{a^2-u^2}}\,du = \sin^{-1}\frac{u}{a} + C$

17. $\int \frac{1}{a^2+u^2}\,du = \frac{1}{a}\tan^{-1}\frac{u}{a} + C$

18. $\int \frac{1}{a^2-u^2}\,du = \frac{1}{2a}\ln\left|\frac{u+a}{u-a}\right| + C$

19. $\int \frac{1}{u\sqrt{u^2-a^2}}\,du = \frac{1}{a}\sec^{-1}\left|\frac{u}{a}\right| + C$

Power rule!

CALCULUS

$\boxed{D_x x^r = rx^{r-1}}$	$D_x\lvert x\rvert = \frac{\lvert x\rvert}{x}$
$D_x \sin x = \cos x$	$D_x \cos x = -\sin x$
$D_x \tan x = \sec^2 x$	$D_x \cot x = -\csc^2 x$
$D_x \sec x = \sec x \tan x$	$D_x \csc x = -\csc x \cot x$
$D_x \sinh x = \cosh x$	$D_x \coth x = -\operatorname{csch}^2 x$
$D_x \cosh x = \sinh x$	$D_x \operatorname{sech} x = -\operatorname{sech} x \tanh x$
$D_x \tanh x = \operatorname{sech}^2 x$	$D_x \operatorname{csch} x = -\operatorname{csch} x \coth x$
$D_x \ln x = \frac{1}{x}$	$D_x \log_a x = \frac{1}{x \ln a}$
$\boxed{D_x e^x = e^x}$	$D_x a^x = a^x \ln a$
$D_x \sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$	$D_x \cos^{-1} x = \frac{-1}{\sqrt{1-x^2}}$
$D_x \tan^{-1} x = \frac{1}{1+x^2}$	$D_x \sec^{-1} x = \frac{1}{\lvert x\rvert\sqrt{x^2-1}}$