Math 480 HW10 2022. Due Wed., Nov. 16. 1 page: problems 1)-4).
Q10 on Friday Nov. 18 is on Markov chains. See Exam 3 review 66)-80).
The final is Wednesday, December 14, 2:45-4:45.

$$\boldsymbol{P} = \begin{bmatrix} & H & S & D \\ H & 0.7 & 0.2 & 0.1 \\ S & 0.3 & 0.5 & 0.2 \\ D & 0.0 & 0.0 & 1.0 \end{bmatrix}$$

1) Suppose the above transition matrix is for a homogeneous Markov chain with states healthy (H), sick (S), and dead (D). Assume each insured is initially healthy. a) Find $\boldsymbol{\pi}_1$. b) Find $\boldsymbol{\pi}_2$. Hint: $\boldsymbol{\pi}_0 = [1 \;\; 0 \;\; 0]$. Find $\boldsymbol{\pi}_1 = \boldsymbol{\pi}_0 \boldsymbol{P}$. Then $\boldsymbol{\pi}_2 = \boldsymbol{\pi}_1 \boldsymbol{P}$.

2) Suppose

$$\boldsymbol{P}^{(1)} = \begin{bmatrix} 0.6 & 0.3 & 0.1 \\ 0.2 & 0.5 & 0.3 \\ 0.0 & 0.0 & 1.0 \end{bmatrix} \quad \text{and} \quad \boldsymbol{P}^{(2)} = \begin{bmatrix} 0.4 & 0.4 & 0.2 \\ 0.0 & 0.4 & 0.6 \\ 0.0 & 0.0 & 1.0 \end{bmatrix}.$$

Suppose the process has states 1, 2, and 3. If the process begins in State 2, what is the probability that the process will be in State 1 after 2 steps? Hint: Want π_{12} where $\boldsymbol{\pi}_2 = (\pi_{12}, \pi_{22}, \pi_{32})$. Find $\boldsymbol{\pi}_1$ then find $\boldsymbol{\pi}_1 \boldsymbol{c}_1$ where $\boldsymbol{c}_1$ is the first column of $\boldsymbol{P}^{(2)}$. (Note that $\boldsymbol{\pi}_1$ is a row vector.)

3) Ross Problem 10.1. Let $\{Z(t), t \geq 0\}$ be a standard Brownian motion. What is the distribution of $Z(s) + Z(t)$ for $s \leq t$?

Hint: $Z(s) + Z(t) = 2Z(s) + Z(t) - Z(s)$ where $Z(s) \perp\!\!\!\perp Z(t) - Z(s)$, $Z(s) \sim N(0, s)$, and $Z(t-s) \sim N(0, t-s)$. Hence $2Z(s) \sim N(0, 4s)$. If $X \sim N(\mu_1, \sigma_1^2) \perp\!\!\!\perp Y \sim N(\mu_2, \sigma_2^2)$, then $X + Y \sim N(\mu_1 + \mu_2, \sigma_1^2 + \sigma_2^2)$.

R Problem The *R* code for Math 480 homework is at (http://parker.ad.siu.edu/Olive/M480Rhw.txt). Copy and paste the source command near the top of this file into *R* to get the *R* programs needed for the homework. See HW4, HW7, and HW8. HW7 tells how to copy and paste *R* output into *Word*.

4a) Copy and paste the *R* command for this part into *R*. This simulates a sample path of a standard Brownian motion on the interval [0,100] (in that the simulated stochastic process converges to a standard Brownian motion when n is large and the program uses $n = 1000000$). Hit the up arrow several times to see a few sample paths. Then include the plot in *Word*.

b) This plot simulates 1000 standard Brownian motion paths on [0,100] using $n = 999$. Then $Z_i = Z(500) \approx N(0, 50)$ is extracted for $Z_1, ..., Z_{1000}$. Then a histogram is made. The histogram should be mound shaped with about 99.7% of the values between -21.2 and 21.2. The plot may use values -20 to 20 on the horizontal scale. The program takes a few seconds because it plots the 1000 sample paths. Make sure you hit Enter to get the histogram. Include the histogram in *Word*.