Math 480 HW11 2022. Due Wed. Nov. 30. Two pages, problems 1)-6). Q10 on Friday Nov. 18 is on Markov Chains and Poisson processes. Q11 on Friday Dec. 2 is on Brownian motion, simulation, normal approximation to a histogram. See Exam 3 review. Exam 3 is Wednesday, Dec. 7. You will get your grades out of 700 points on Friday, Dec. 9. The final is Wednesday, December 14, 2:45-4:45.

1) Suppose $A(t)$ follows an arithmetic Brownian motion with drift $\mu = 5$ and volatility $\sigma = 2$. If $A(3) = 35$, calculate the probability that $A(5) \leq 48$.

Hint: see problem done in class and exam 3 review 90): $W \sim A(t+s)|A(t) \sim N(A(t) + \mu s, \sigma^2 s)$. Here $t = 3$ and $s = 2$. Want $P(W \leq 48)$.

2) Suppose $X \sim$ Weibull($\theta = 100, \tau = 1$). Simulate two values of x_i from this distribution if $u_1 = 0.69$ and $u_2 = 0.13$. Hint: $x_i = F^{-1}(u_i)$ where $F^{-1}(u) = \theta[-\log(1-u)]^{1/\tau}$.

3) Normal approximation to the pmf of a binomial distribution (correction for continuity): Suppose that the probability that a patient recovers from a certain blood disease is 0.4. Find the approximate probability that at least 36 of the next 100 patients who contract this disease survive. (Hint: Let Y be the number of patients who recover, then Y is binomial($n = 100, p = 0.4$). Let X be a normal RV with mean $\mu = np$ and SD $\sigma = \sqrt{np(1-p)}$ and find $P(X \geq 35.5)$.)

Also see exam 3 review 95).

4) The data below are a sorted random sample of size 11 from the R Poisson($\lambda = 10$) random number generator. Find shorth(7).

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5    5    7    8    8    8   11   12   14   16   18
```

a) Find shorth(7).

b) Find $\hat{x}_{0.8}$, the estimator of the 80th population percentile.

Hint: see exam 3 review 98) and 97).

R Problems The R code for Math 480 homework is at (http://parker.ad.siu.edu/Olive/M480Rhw.txt). Copy and paste the source command near the top of this file into R to get the R programs needed for the homework. See HW4, HW7, and HW8. HW7 tells how to copy and paste R output into *Word*.

5) This problem consider the Cauchy(0,1) random number generator. One million Cauchy(0,1) pseudo random variables are generated. The problem will compare the population 2.5% and 97.5% percentiles with two estimates. Either write down the two numbers produced for each part, or include the numbers in *Word*.

a) Copy and paste the R command for this part into R. This code computes the two population 2.5th and 97.5th percentiles $F^{-1}(0.025)$ and $F^{-1}(0.925)$.

b) Copy and paste the R command for this part into R. This code computes the two sample percentiles that estimate the population 2.5th and 97.5th percentiles.

c) Copy and paste the R command for this part into R. This code computes the shorth estimator of the two population 2.5% and 97.5% percentiles.

6) Copy and paste the *R* command for this part into *R*. The function generates 201 pseudo uniform(0,1) random variables. $(u_1, ..., u_{200})$ and plots u_{i+1} vs. u_i for $i = 1, ..., 100$. Include the plot in *Word*. The 200 plotted points should fill the unit square without any pattern.

(Following Ross p. 646, the `rng` function computes $x_{n+1} = (69069 * x_n + 1)$ %% (2^{32}) and takes $u_i = x_i/2^{32}$. The *R* modulo function %% computes the remainder:
e_1 %% $e_2 = e_1 - (\lfloor e_1/e_2 \rfloor)e_2$.) The *R* function `runif` is a better function.