Math 480 HW 3 2022. Due Wed. Sept. 14. **2 pages, 8 problems**

1) Let Y be a random variable with $p(y)$ given by the accompanying table. Find $E(Y)$, $E(1/Y)$, $E(Y^2-1)$ and $V(Y)$.

y	1	2	3	4
p(y)	0.4	0.3	0.2	0.1

comment: The short cut formula $V(Y) = E(Y^2) - [E(Y)]^2$ on p. 41 reduces the amount of work. Note that $E(Y^2) = E(Y^2 - 1) + 1$ and $E(Y^2 - 1) = E(Y^2) - 1$.

2) A single fair die is tossed once. Let Y be the number facing up. Find the expected value and variance of Y.

comment: The short cut formula $V(Y) = E(Y^2) - [E(Y)]^2$ on p. 41 reduces the amount of work.

3) Ross problem 2.8 on p. 80. Suppose the cdf of X is given by

$$F(x) = \begin{cases} 0, & x < 0 \\ 0.5, & 0 \le x < 1 \\ 1, & 1 \le x < \infty \end{cases}$$

What is the pmf of X?

comment: Just give $p(x)$ for the two values of x with $p(x) > 0$ that occur where the cdf $F(x)$ jumps with $p(x) = F(x) = F(x-)$.

4) Ross problem 2.11 on p. 80 modified. A ball is drawn form an urn containing three white and three black balls. After the ball is drawn, it is replaced an another ball is drawn. This goes on indefinitely. Let X count the number of white balls in the first four draws.

a) What is the probability that of the first four balls drawn, exactly two are white.

b) Find $E(X)$.

c) Find $V(X)$.

comment: Let X count the number of white balls drawn in the first 4 draws. Then $X \sim bin(n = 4, p = 0.5)$. For a) find $P(X = 2) = p(2)$ and the answer to a) is in the back of the book. Formulas p. 2 0) i) of exam 1 review for the binomial distribution are useful.

5) Ross problem 12 on p. 80 modified. Consider a multiple choice exam with three possible answers for each of 5 questions. Let X count the number of correct answers if the student guesses.

a) What is the probability that the student gets 4 or more correct answers just by guessing.

b) Find $E(X)$.

c) Find $V(X)$.

comment: $X \sim bin(n = 5, p = 1/3)$. For a), want P(X is at least 4) $= P(X \ge 4) = p(4) + p(5)$. Also see the comment for problem 4).

6) A particular concentration of a chemical found in polluted water has been found to be lethal to 20% of the fish that are exposed to the concentration for 24 hours. Twenty fish are placed in a tank containing this concentration of chemical in water. Find the probability that at most 16 survive.

comment: Let Y be the number that survive. Then $Y \sim bin(n = 20, p)$. Note that the probability p that a randomly selected fish survives is obtained using the complement rule. Also P(at most 16) $= P(Y \leq 16) = 1 - P(Y > 16) = 1 - P(Y \geq 17) = 1 - p(17) - p(18) - p(19) - p(20)$.

7) Let Y denote a Poisson random variable with mean $\lambda = 2$. a) Find $P(Y = 4)$. b) Find V(Y).

comment: Formulas p. 2 0) k) of exam 1 review for the Poisson distribution are useful.

8) The number of knots in a type of wood has a Poisson distribution with an average of 1.5 knots in 10 cubic feet of wood. Find the probability that a 10 cubic block of wood has at most one knot.

comment: Let Y count the number of knots. Then P(Y is at most one) $= P(Y \leq 1) = p(0) + p(1)$. Also see the comment for 7).