

Math 480 HW 4 2022. Due Wed. Sept. 21.

Exam 1 is Wed. Sept. 28 and is like quizzes 1-4. **2 pages, 6 problems**

1) Ross Problem 2.38 modified. Suppose X has pdf $f(x) = c e^{-2x}$ for $x \geq 0$, and $f(x) = 0$ for $x < 0$. a) Find c . b) Find $P(X > 2)$. c) Find $F(x)$ for $x > 0$. ($F(x) = 0$ for $x \leq 0$.) Hint: $\int_0^\infty f(x)dx = 1$, $P(X > 2) = \int_2^\infty f(x)dx$ and $F(x) = \int_0^x f(t)dt$ for $x > 0$.

2) Suppose that the probability density function for a random variable Y is given by

$$f(y) = \begin{cases} c y, & \text{if } 0 \leq y \leq 5 \\ 0, & \text{otherwise.} \end{cases}$$

a) Find c .

b) Find $E(Y)$.

c) Find $V(Y)$. Hint: Find $E(Y^2)$ and use the short cut formula.

3) Suppose that Y is a random variable with distribution function

$$F(y) = 1 - e^{-y}$$

for $y > 0$ and that $F(y) = 0$ otherwise.

a) Find $f(y)$ for $y > 0$. (Note that $f(y) = 0$ for $y < 0$.)

b) Find $P(Y > 6) = 1 - P(Y \leq 6) = 1 - F(6)$.

4) If Y has probability density function

$$f(y) = \begin{cases} (1/2)(2 - y), & \text{if } 0 \leq y \leq 2 \\ 0, & \text{otherwise,} \end{cases}$$

find the mean and variance of Y .

comment: Use the short cut formula to find $V(Y)$.

5) If Y is a continuous random variable with mean μ and variance σ^2 and if a and b are constants, use $E(c) = c$, $E[cg(Y)] = cE[g(Y)]$ and $E[\sum_{i=1}^k g_i(Y)] = \sum_{i=1}^k E[g_i(Y)]$ to prove the following.

a) $E(aY + b) = aE(Y) + b = a\mu + b$.

b) $V(aY + b) = a^2V(Y) = a^2\sigma^2$.

comment: For b) let $W = aY + b$ and use a). Using the definition

$V(Y) = E(Y - E[Y])^2$ is easier than using the short cut formula.

An R problem is on the back. R is free software available from (<https://cran.r-project.org/>), and is heavily used in statistics research and actuarial mathematics. SAS is the industry standard for statistics but is expensive. So a small company will often have R instead of SAS . R is also used in probability research and financial mathematics. See (<http://parker.ad.siu.edu/Olive/rch17.pdf>) for some more information on R . The software is also available in the Math computer lab in Neckers 258. Computers 11-25 likely have R with a desktop icon with an R symbol.

Every fall, the initial login to one of these computers can take 10 minutes, and that is 10 minutes for each different computer. The computer on button is in the upper left corner while the monitor on button is in the lower right corner. You may need to press a computer keyboard key to get the login and password bars to appear.

The computer lab login is like logging into salukinet. If necessary, hit Ctrl, enter your AD\siu8... (dawg tag) and your password. Left click the lower left icon, or near the lower left icon, search for R.

Click the internet icon and search for David Olive. His personal page (<http://parker.ad.siu.edu/Olive/Personal.html>) has a Links for Students which has information on Math 480, Math and Statistics Texts, and the SIU Actuarial program. R code for Math 480 homework (HW) is on this page and at (<http://parker.ad.siu.edu/Olive/M480Rhw.txt>). The source code near the top of this file gets R programs needed for some of the homework into R quickly. You can copy and paste the R code for each problem part.

In the future, having a flashdrive to save R output may be useful.

For brand name distributions, `pnorm(q,mean=0,sd=1)` gives $F(q)$ for the normal $N(0, 1)$ distribution, while `pnorm(q,mean=100,sd=15)` gives $F(q)$ for the normal $N(\mu = 100, \sigma^2 = 15^2)$ distribution. The command

```
qnorm(p,mean=100,sd=15)
```

gives the percentile x_p such that $F(x_p) = p$ if $F(x) = P(X \leq x)$ and $X \sim N(100, 15^2)$. Enter the command `?Distributions` or `help(Distributions)` to see some of the other brand name distributions with simple R commands.

My office is right next to the Math computer lab. Come in for help if the R problem takes more than a few minutes to do (not counting the initial login).

6) IQ tests are often standardized to have mean $\mu = 100$ and standard deviation $\sigma = 15$, and are approximately normal. A 2018 survey said women find intelligence at the 90th percentile an attractive trait in a date, but not at the 99th percentile or higher because such men are socially awkward and high maintenance.

a) Copy and paste or type the following command in R to find the 90th percentile of such IQ tests.

```
qnorm(p=0.9,mean=100,sd=15)
```

b) Copy and paste or type the following command in R to find the 99th percentile of such IQ tests.

```
qnorm(p=0.99,mean=100,sd=15)
```

Write the values for a) and b) on your homework, you do not need to print out the output.