Math 480 HW 6 2022. Due Wed. Oct. 12. Q6 on Friday Oct. 14, is like HW6. The final is Wednesday, Dec. 14, 2:45-4:45.

1) Suppose that the joint pmf of Y_1 and Y_2 is given by the table below. a) Are Y_1 and Y_2 independent? Why? comment: The support is not a cross product. b) Find $\text{Cov}(Y_1, Y_2)$.

		y_2		
$p(y_1, y_2)$		0	1	2
	0	1/9	2/9	1/9
y_1	1	2/9	2/9	0
	2	1/9	0	0

2) Suppose that the joint pmf of Y_1 and Y_2 is given by the table below. a) Are Y_1 and Y_2 independent? Why? b) Find $\text{Cov}(Y_1, Y_2)$.

		y_1		
$p(y_1, y_2)$		0	1	total
	0	0.35	0.15	0.5
y_2	1	0.14	0.06	0.2
	2	0.21	0.09	0.3
	total	0.7	0.3	1.00

3) Let Y_1 and Y_2 have joint pdf

$$f(y_1, y_2) = \begin{cases} 4y_1y_2, & \text{if } 0 \le y_1 \le 1, \ 0 \le y_2 \le 1 \\ 0, & \text{otherwise.} \end{cases}$$

a) Are Y_1 and Y_2 independent? Why? b) Find $\text{Cov}(Y_1, Y_2)$.
Hint: See the second to last paragraph on p. 4 of exam 2 review.

4) Let Y_1 and Y_2 have joint pdf

$$f(y_1, y_2) = \begin{cases} 6(1 - y_2), & \text{if } 0 \le y_1 \le y_2 \le 1 \\ 0, & \text{otherwise.} \end{cases}$$

The marginal pdfs are

$$f_{Y_1}(y_1) = 3(1 - y_1)^2, \text{ for } 0 \le y_1 \le 1$$

and

$$f_{Y_2}(y_2) = 6y_2(1 - y_2), \text{ for } 0 \le y_2 \le 1$$

where the marginal pdfs are zero elsewhere.
a) Find $E(Y_1)$.
b) Find $E(Y_2)$.
c) Find $Cov(Y_1, Y_2)$. (The dy_1dy_2 double integral is easier.)
d) Are Y_1 and Y_2 independent? Explain.

x	-2	-1	0	1	2
p(x)	0.1	0.3	0.3	0.2	0.1

5) Let the discrete random variable X have a probability mass function given by the table above. Find the pmf of $Y = 4X^2 + 3$.

6) Let X be a random variable with pdf

$$f(x) = 2x \quad \text{where} \quad 0 < \text{x} < 1.$$

Let $Y = 8X^3$ and find the pdf of Y using the method of transformations. Do not forget to include the support of Y.

7) Suppose that X is a random variable with pdf

$$f(x) = \frac{1}{\lambda} x^{\frac{1}{\lambda} - 1},$$

where $0 < x \leq 1$ and $\lambda > 0$. Let $Y = -\log(X)$ and find the pdf of Y. Do not forget to include the support of Y. Recall that $\log(x)$ is the natural logarithm in this class.

8) Suppose that the moment generating function (mgf) of a random variable Y is

$$\phi(t) = \frac{1}{1 - \lambda\, t}$$

where $\lambda > 0$ is a **known constant.** Using the mgf $\phi(t)$, find $\phi'(t)$ and $E(Y)$.

9) Let X be a random variable with pgf $P_X(z) = \dfrac{1}{c} \displaystyle\sum_{n=1}^{3} \frac{z^n}{10^n}$.

a) Find c.
b) Find $P(X = 3)$.
c) Find $P'_X(z)$.
d) Find $E(X)$.

10) Suppose that $Y_1, ..., Y_n$ are independent random variables where $E(Y_i) = r_i/p$, $V(Y_i) = \dfrac{r_i(1-p)}{p^2}$ and the moment generating function of Y_i is

$$\phi_{Y_i}(t) = \left[\frac{pe^t}{1 - (1-p)e^t} \right]^{r_i}$$

for any real t. Let $U = \displaystyle\sum_{i=1}^{n} Y_i$.

a) Find E(U).
b) Find the variance V(U) of U.
c) Find the moment generating function $\phi_U(t)$ of U. (Simplify the product.)