Math 480 HW 7 2022. Due Wed. Oct. 19. Q7 on Friday Oct. 21. The final is Wednesday, Dec. 14, 2:45-4:45.

1) Suppose $X_1$ and $X_2$ are independent, that $X_1 \sim$ Poisson($\lambda = 1$), and $X_2 \sim$ Poisson($\lambda = 2$). Consider $U = X_1 + X_2$.
a) What is the distribution of $U$?
b) What is the mgf of $U = X_1 + X_2$?
c) For integer $k \geq 0$, find $P(X_1 + X_2 = k)$.
d) Find $P(X_1 + X_2 \geq 2)$.
e) Let $Y = 2X_1 - 5X_2$. Find i) the mean $Y$ and ii) the variance of $Y$.

2) Suppose $X$ has a distribution with mgf $\phi_X(t) = e^{5t+10t^2}$.
a) What is the distribution of $X$?
b) Let $X_i$, $i \geq 1$ be iid with the same distribution as $X$.
i) What is the distribution of $\sum_{i=1}^{10} X_i$?
ii) State or find the limit $\lim_{n\to\infty} \frac{1}{n}\sum_{i=1}^{n} X_i$. Justify your answer (which theorem applies)?

3) Let $Y_1$ and $Y_2$ have joint pdf

$$f(y_1, y_2) = \begin{cases} 6(1-y_2), & \text{if } 0 \leq y_1 \leq y_2 \leq 1 \\ 0, & \text{otherwise.} \end{cases}$$

The marginal pdfs are

$$f_{Y_1}(y_1) = 3(1-y_1)^2, \text{ for } 0 \leq y_1 \leq 1$$

and

$$f_{Y_2}(y_2) = 6y_2(1-y_2), \text{ for } 0 \leq y_2 \leq 1$$

where the marginal pdfs are zero elsewhere.

a) Find the conditional pdf of $Y_1$ given $Y_2 = y_2$.
b) Find the conditional pdf of $Y_2$ given $Y_1 = y_1$.
For a), be very careful about the domain of $y_1$.
For b), be very careful about the domain of $y_2$.

4) Let $Y_1$ and $Y_2$ have joint pdf

$$f(y_1, y_2) = \begin{cases} 4y_1y_2, & \text{if } 0 \leq y_1 \leq 1, \ 0 \leq y_2 \leq 1 \\ 0, & \text{otherwise.} \end{cases}$$

The marginal pdfs are

$$f_{Y_1}(y_1) = 2y_1, \text{ for } 0 \leq y_1 \leq 1$$

and

$$f_{Y_2}(y_2) = 2y_2, \text{ for } 0 \leq y_2 \leq 1$$

where the marginal pdfs are zero elsewhere. Find the conditional pdf of $Y_2$ given $Y_1 = y_1$. Hint: if $Y_1 \perp\!\!\!\perp Y_2$, then the conditional pdf is the marginal pdf, but show using the formula.

5) Suppose the joint pmf is table below where

$$Y_1 = \begin{cases} 0, & \text{if no belt used} \\ 1, & \text{if adult belt used} \\ 2, & \text{if car-seat belt used.} \end{cases} \quad \text{and } Y_2 = \begin{cases} 0, & \text{if child survived} \\ 1, & \text{otherwise} \end{cases}$$

| $p(y_1, y_2)$ | | $y_2$ 0 | 1 | total |
|---|---|---|---|---|
| | 0 | 0.38 | 0.17 | 0.55 |
| $y_1$ | 1 | 0.14 | 0.02 | 0.16 |
| | 2 | 0.24 | 0.05 | 0.29 |
| | total | 0.76 | 0.24 | 1.00 |

a) Give the marginal probability mass function for $Y_2$.
b) Give the conditional probability mass function for $Y_1$ given $Y_2 = 0$.

6) Assume that $\overline{Y}$ is computed from a random sample of size $n = 4$ drawn from a highly skewed population with mean $\mu = 12$ and standard deviation $\sigma = 1.6$. If possible, find $P(11.8 < \overline{Y})$.

7) Assume that $\overline{Y}$ is computed from a random sample of size $n = 4$ drawn from a normal population with mean $\mu = 12$ and standard deviation $\sigma = 1.6$. If possible, find $P(11.8 < \overline{Y})$.

*R Problem*: If you email me the *R* output, put your name on the *Word* file. As described on HW4, the *R* code for Math 480 homework is at (http://parker.ad.siu.edu/Olive/M480Rhw.txt). Copy and paste the source command near the top of this file into *R* to get the *R* programs `cltsim` and `cltsim2` needed for the homework. A histogram is a pdf estimator.

Near the lower left icon, search for *Word*, etc. To get a plot in *Word*: click on the plot in *R* and press the *Ctrl* and *c* keys as the same time. Then click on *Word* and press the *Ctrl* and *v* keys as the same time (or use the menu command "Paste" in *Word*). **My office is right next to the Math computer lab. Come in for help** if the *R* problem takes more than a few minutes to do.

8) a) Copy and paste the *R* commands for this part into *R*. The function is used to illustrate the central limit theorem when the data $Y_1, ..., Y_n$ are iid from an exponential distribution. The function generates a data set of size $n$ and computes $\overline{Y}_1$ from the data set. This step is repeated $nruns = 1000$ times. The output is a vector $(\overline{Y}_1, \overline{Y}_2, ..., \overline{Y}_{1000})$. A histogram of these means should resemble a symmetric normal density once $n$ is large enough. The *R* commands will plot 4 histograms with $n = 1, 5, 25$ and 200. Save the plot in *Word*.

b) Explain how your plot illustrates the central limit theorem.

c) Copy and paste the *R* commands for this part into *R*. Now $Y_1, ..., Y_n$ are iid N(0,1) and $\overline{Y} \sim N(0, 1/n)$. The *R* commands will plot 4 histograms with $n = 1, 5, 25$ and 200. Save the plot in *Word*.

d) Explain how your plot illustrates the central limit theorem. (The horizontal scale should decrease as $n$ increases.)