Math 480 HW 8 2022. Due Wed. Oct. 26. Q8 on Friday Oct. 28. Exam 2 is Wednesday, Nov. 2. Old Q9 is useful for HW 8. The final is Wednesday, Dec. 14, 2:45-4:45.

1) Suppose that the conditional distribution of $Y|\Lambda$ is a Poisson(Λ) distribution and that the random variable Λ has a gamma (k, λ) distribution.

a) Find E(Y).
b) Find V(Y). Hint: See examples in class and E2 rev 48).

2) Heart/Lung transplant claims in 2017 follow a Poisson process where the number of claims $N(t)$ occur at a rate of $\lambda = 1$ per month. As of the end of January, 2007 no transplant claims have arrived. Calculate the probability that at least three Heart/Lung transplant claims will have arrived by the end of March, 2007. Hint: want the number of claims in 2 months (Feb. and March). Let W = number of claims in 2 months. Want $P(W \geq 3)$.

3) Suppose $N(t)$ counts the number of events at a Poisson rate of 0.02 per day. Find the probability that no events occur in a 30 day period.

4) An underwriter receives life insurance applications between time $t = 0$ and $t = 2$ in a nonhomogeneous Poisson process at a rate of $\lambda(t) = 2t - t^2$. Calculate the probability of receiving exactly 3 applications between $t = 0$ and $t = 2$.

5) Suppose $X(t) = \sum_{i=1}^{N(t)} Y_i$ follows a compound Poisson process where the number of claims $N(t)$ occur at a rate of $\lambda = 10$ per day and the claim severity distribution Y_i is exponential with mean $= \theta = 15000$. (Note that $Y_i \sim EXP(\beta)$ with $\beta = 1/\theta$.)

a) Find $E[X(t)]$.

b) Find $SD[X(t)]$. Hint: $SD[X(t)] = \sqrt{V[X(t)]}$.

c) Suppose a claim is a large loss if its severity is greater than 50000. Find the probability that a claim is a large loss. Hint: Want $P(Y_1 > 50000) = 1 - F(50000) = \exp(-50000\beta) = \exp(-50000/\theta)$.

d) Find the probability of exactly 9 large losses in a 30 day period. Hint: $N_L(t)$ is a Poisson process with rate $\lambda_L = 10P(Y_1 > 50000)$ per day. So $W \sim Poisson(\lambda_L 30)$. Want $P(W = 9)$.

R Problems: If you email me the *R* output, put your name on the *Word* file. As described on HW4, the *R* code for Math 480 homework is at (http://parker.ad.siu.edu/Olive/M480Rhw.txt). **Copy and paste the source command near the top of this file into** *R* to get the *R* programs needed for the homework. See HW4 and HW7.

6) Copy and paste the *R* commands for this part into *R*. The function is used to make sample paths for the random walk. Save the plot in *Word*. Near the lower left of the keyboard are four arrows. If you hit the up arrow, the most recent *R* command appears, hit the up arrow twice and the second to last *R* command appears, etc. This can save typing and pasting. Hit the up arrow several times to see several sample paths. You may need to click on "R Console window" for the command to appear. Then save the plot in *Word*.

The plot is of $Y_t = Y_{t-1} + e_t = y_0 + \sum_{i=1}^t e_i$ for $t = 1, ..., 100$ where $y_0 = 1$ and the e_i are iid $EXP(1)$. The line with slope 1 and intercept 1 is the expected value line $E(Y_t) = 1 + tE(e_i) = 1 + t$.

7) Copy and paste the *R* commands for this part into *R*. The function is used to make sample paths for the Poisson process with $n = 100$ events and rate $\lambda = 1$. Hit the up arrow several times to see several sample paths. Then save the plot in *Word*.

The step function in the plot is $N(t)$ where $N(t_i^*) = i$ since the ith event occurs at the waiting time t_i^* to the ith event where $0 < t_1^* < \cdots < t_{100}^*$. The waiting time to the ith event $t_i^* = S_i = \sum_{j=1}^i X_j$ where $X_j \sim EXP(\lambda = rate)$ with $E(X_j) = 1/\lambda$. The X_j are iid. Hence $E(t_i^*) = E(S_i) = i/\lambda$ is about the time t_i^* when $N(t_i^*) = i$. Note that if the rate is $\lambda = 60$ per hour, then time is in hours and an event should occur about every (1/60)th of an hour or once a minute. Note that $\lambda i/\lambda = i \approx N(t)$ at $t \approx i/\lambda$ for $i = 1, 2, ..., n, ...$ Hence the sample path should roughly follow the solid line $N_p = \lambda t$ that has intercept 0 and slope = rate = λ, and $\lambda = 1$ for the simulation.