Math 480 Exam 1 Fall 2018 Name_____

YOU ARE BEING GRADED FOR WORK, NOT JUST THE FINAL ANSWER

y	2	3	4	5	6
p(y)	0.41	0.32	0.14	?	?

e 1) Let the discrete random variable Y be the number of years a randomly selected SIU alumni took to graduate if the alumni was a transfer student in 1993 who graduated in six or fewer years after transferring. The table above displays the approximate probability distribution of Y. What is the probability that Y is greater than 4? $= P(Y>4)$

$= 1 - P(Y \le 4) = 1 - .41 - .32 - .14 = \boxed{0.13}$

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y	3	4	5	6
p(y)	0.03	0.37	0.47	0.13

2) Let the discrete random variable Y be the number of years a randomly selected SIU alumni took to graduate (for 1993 entering Freshman who graduated in six years). The table above displays the approximate probability distribution of Y.

a) Find E(Y). $= \sum y P(y) = 3(.03) + 4(.37) + 5(.47) + 6(.13)$

$= \boxed{4.70}$

b) Find $E(Y^2)$. $= \sum y^2 P(y) = 9(.03) + 16(.37) + 25(.47) + 36(.13)$

$= \boxed{22.62}$

c) Find the standard deviation of Y. $= \sqrt{V(Y)} = \sqrt{E(Y^2) - (EY)^2}$

$= \sqrt{22.62 - (4.7)^2} = \sqrt{0.53} = \boxed{0.7280}$

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(or $V(X) = (3-4.7)^2(.03) + (4-4.7)^2(.37) + (5-4.7)^2(.47) + (6-4.7)^2(.13)$

$= 0.53$)

3) The table below shows whether the accused murderer received a death sentence based on the race of the victim. Let D be the event that the accused murderer received a death sentence. Let W be the event that the victim was white.

death sentence	white victim	black victim	total
yes	108	20	128
no	865	1482	2347
total	973	1502	2475

a) What is the probability that the victim was white?

$P(W) = \frac{973}{2475} = 0.3931$

b) What is the probability that the victim was white given that the accused murderer received a death sentence?

$P(W|D) = \frac{108}{128} = \frac{108/2475}{128/2475} \approx 0.8437 \approx .8438 = \frac{P(W \cap D)}{P(D)}$

$.84375$

c) Are the events W and D independent? Explain.

NO $P(W|D) = .8437 \neq P(W) = .3931$

or $P(W \cap D) = \frac{108}{2475} = .04364 \neq P(W)P(D) = .3931 \frac{128}{2475} = .3931(.0517)$

$= 0.02033$

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4) Suppose that a multiple choice quiz has 15 questions, each question with 5 choices for the answer, and each question has exactly one correct answer. Suppose that a student randomly guesses the answer for each of the 15 questions. Let the binomial random variable X count the number of questions the student got correct. Find the probability that the student got at least one question correct. Simplify.

$X \sim bin(n=15, p=\frac{1}{5})$ $\quad P(X \geq 1) =$

$1 - P(X=0) = 1 - \binom{15}{0}\left(\frac{1}{5}\right)^0\left(\frac{4}{5}\right)^{15} = 1 - \left(\frac{4}{5}\right)^{15} = 1 - (0.8)^{15}$

$= 1 - 0.03518 = 0.9648$

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$\int_1^\infty 4y^{-5}\,dy = \frac{4y^{-4}}{-4}\Big|_1^\infty = 0+1$ $\quad \frac{d}{dy}y^{-4} = -4y^{-5}$

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5) Suppose that the probability density function for a random variable Y is given by

$$f(y) = \begin{cases} 4\,y^{-5}, & \text{if } y \geq 1 \\ 0, & \text{otherwise.} \end{cases}$$

Note that the support is $(1, \infty)$ not $(0, \infty)$.

a) Find $E(Y)$. $= \int y f(y)\,dy = \int_1^\infty y\,4y^{-5}\,dy = 4\int_1^\infty y^{-4}\,dy = \frac{4y^{-3}}{-3}\Big|_1^\infty = 0+\frac{4}{3}$

$= \boxed{\frac{4}{3} = 1.3333}$

b) Find $V(Y)$.

$E(Y^2) = \int y^2 f(y)\,dy = \int_1^\infty y^2\,4y^{-5}\,dy = 4\int_1^\infty y^{-3}\,dy = \frac{4y^{-2}}{-2}\Big|_1^\infty = 0+2 = \underline{2}$

$V(Y) = E(Y^2) - (EY)^2 = 2 - \left(\frac{4}{3}\right)^2 = \frac{18-16}{9} = \boxed{\frac{2}{9} = 0.2222}$

c) Find $F(y)$. $= \int_1^y 4t^{-5}\,dt = \frac{4t^{-4}}{-4}\Big|_1^y = \begin{cases} 1 - y^{-4} & 1 < y < \infty \\ 0 & y \leq 1 \end{cases}$

check: $\frac{d}{dy}F(y) = -(-4)\,y^{-5} = 4y^{-5}$

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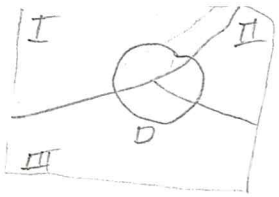
$P(D) = P(I \cap D) + P(II \cap D) + P(III \cap D)$



6) Three machines, I, II, and III, manufacture 0.35, 0.20, and 0.45 of the total production in a manufacturing plant, respectively. The proportion of defective items produced by I, II, and II is 0.01, 0.018, and 0.02, respectively.

a) What is the probability that a randomly chosen item is defective?

$P(D) = P(I)\,P(D|I) + P(II)\,P(D|II) + P(III)\,P(D|III)$

$= .35(.01) + .2(.018) + .45(.02) = \boxed{0.01610}$

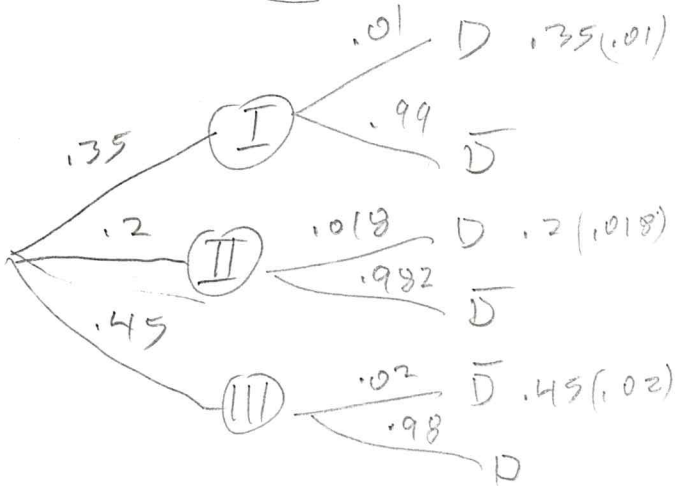
12/14 got it

b) Given that a randomly chosen item is defective, find the probability that the item came from machine I.

Bayes rule $P(I|D) = \dfrac{P(I \cap D)}{P(D)}$

$= \dfrac{P(I)\,P(D|I)}{P(I)\,P(D|I) + P(II)\,P(D|II) + P(III)\,P(D|III)}$

$= \dfrac{.35(.01)}{.01610} = \dfrac{.0035}{.01610} = \boxed{0.2174}$



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