Math 480 Exam 2, Fall 2018 Name____________________

1) Suppose that the joint pmf of Y_1 and Y_2 is $p(y_1, y_2)$ is tabled below.

$p(y_1,y_2)$		y_2 = 0	1	2	3	$P_{Y_1}(y_1)$
y_1	0	2/12	2/12	1/12	1/12	$6/12 = \frac{1}{2}$
	1	2/12	2/12	0/12	0/12	$4/12 = \frac{1}{3}$
	2	1/12	1/12	0/12	0/12	$2/12 = \frac{1}{6}$
$P_{Y_2}(y_2)$		$\frac{5}{12}$	$\frac{5}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	

a) Are Y_1 and Y_2 independent? Explain.

no, support is not a cross product

$p(0,0) = \frac{2}{12} \neq P_{Y_1}(0)P_{Y_2}(0) = \frac{6}{12}\frac{5}{12} = \frac{5}{24}$ (no, $cov(Y_1,Y_2) \neq 0$)

b) Find $E(Y_1)$. $\sum y_1 P_{Y_1}(y_1) = (0)\frac{6}{12} + (1)\frac{4}{12} + (2)\frac{2}{12} = \boxed{\frac{8}{12} = \frac{2}{3} = 0.6667}$

c) Find $E(Y_2)$. $= \sum y_2 P_{Y_2}(y_2) = (0)\frac{5}{12} + (1)\frac{5}{12} + (2)\frac{1}{12} + (3)\frac{1}{12}$

$= \frac{10}{12} = \frac{5}{6} = 0.8333$

d) Find $\text{Cov}(Y_1, Y_2)$. $E(Y_1Y_2) = \sum\sum y_1 y_2 \, p(y_1,y_2)$

$= 0 + 0 + 0 + 0$

$+ 0 + 1(1)\frac{2}{12} + 0 + 0$

$+ 0 + 2(1)\frac{1}{12} + 0 + 0 = \frac{4}{12} = \frac{1}{3}$

$\leftarrow \sim 2$

$cov(Y_1,Y_2) = E(Y_1Y_2) - E(Y_1)E(Y_2) = \frac{1}{3} - \frac{5}{6}\frac{2}{3}$

$= \frac{6-10}{18} = \frac{-4}{18} = \boxed{\frac{-2}{9} = -0.2222} = \frac{-32}{144}$

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2) Suppose that the lengths Y of catfish in a river come from a normal distribution with mean $\mu = 19$ and standard deviation $\sigma = 4.5$. Assume that the sample mean $\overline{Y}$ is computed from a sample of size $n = 11$. Find $P(18 \leq \overline{Y} \leq 22)$ if possible.

$\mu_{\bar Y} = 19$, $\sigma_{\bar Y} = \frac{\sigma}{\sqrt{n}} = \frac{4.5}{\sqrt{11}} = 1.3568$

18 19 20 21 22

$$z = \frac{18-19}{4.5/\sqrt{11}} = \frac{18-19}{1.3568} = -0.74, \quad z = \frac{22-19}{4.5/\sqrt{11}} = \frac{22-19}{1.3568} = 2.21$$

-.74 2.21

	.01	.04
-0.7		.2296
2.2	.9864	

$$P(18 \le \bar Y \le 22) = P(-0.74 \le Z \le 2.21)$$

$$= .9864 - .2296 = \boxed{0.7568}$$

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3) Suppose that the lengths Y of catfish in a river come from a highly skewed distribution with mean $\mu = 19$ and standard deviation $\sigma = 4.5$. Assume that the sample mean $\overline{Y}$ is computed from a sample of size $n = 11$. Find $P(18 \leq \overline{Y} \leq 22)$ if possible.

not possible

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4) Suppose $N(t)$ counts the number of events at a Poisson rate of 0.01 per day. Find the probability that no events occur in a 40 day period.

$P(W=k) = \frac{e^{-\mu}\mu^k}{k!}$

$$W = N(40) \sim \text{Poisson}(\lambda t = 0.01(40) = 0.4 = \mu)$$

$$P(W=0) = P(N(40)=0) = \frac{e^{-0.4}(0.4)^0}{0!} = \boxed{e^{-0.4} = 0.6703}$$

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x^2	1	1	4
x	−1	1	2
p(x)	3/27	9/27	15/27

e

5) Let the discrete random variable X have a pmf given by the table above. Find the pmf of $Y = X^2$.

y	1	4
P(y)	$\frac{12}{27} = \frac{4}{9} = 0.4444$	$\frac{15}{27} = \frac{5}{9} = .5556$

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6) Suppose that $Y_1, ..., Y_n$ are independent random variables where $E(Y_i) = V(Y_i) = \lambda_i$ and the moment generating function of Y_i is $\phi_{Y_i}(t) = \exp[(e^t - 1)\lambda_i]$ for any real t. Let $U = \sum_{i=1}^{n} Y_i$.

a) Find E(U). $= \sum_{i=1}^{n} E(Y_i) = \boxed{\sum_{i=1}^{n} \lambda_i}$

b) Find the variance V(U) of U. $\stackrel{ind}{=} \sum_{i=1}^{n} V(Y_i) = \boxed{\sum_{i=1}^{n} \lambda_i}$

c) Find the moment generating function $\phi_U(t)$ of U. $= \prod_{i=1}^{n} \phi_{Y_i}(t) =$

$$\prod_{i=1}^{n} \exp[(e^t - 1)\lambda_i] = \boxed{\exp[(e^t - 1)\sum_{i=1}^{n} \lambda_i]}$$

$$= e^{(e^t-1)\sum_{i=1}^{n} \lambda_i}$$

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$e^{ab} = (e^a)^b$

7) Suppose that the joint pdf of the random variables Y_1 and Y_2 is given by

$$f(y_1, y_2) = y_1 + y_2, \text{ if } 0 \le y_1 \le 1, 0 \le y_2 \le 1$$

and $f(y_1, y_2) = 0$, otherwise.

a) Find the marginal pdf of Y_2. Include the support.

$$f_{Y_2}(y_2) = \int_0^1 y_1 + y_2 \, dy_1 = \frac{y_1^2}{2} + y_2 y_1 \Big|_0^1 = \boxed{\frac{1}{2} + y_2, \quad 0 < y_2 < 1}$$

b) Find $E(Y_2)$. $= \int_0^1 y_2 f_{Y_2}(y_2) \, dy_2 = \int_0^1 y_2 \left(\frac{1}{2} + y_2\right) dy_2 = \int_0^1 \frac{y_2}{2} + y_2^2 \, dy_2$

$$= \frac{y_2^2}{4} + \frac{y_2^3}{3} \Big|_0^1 = \frac{1}{4} + \frac{1}{3} = \boxed{\frac{7}{12} = 0.5833}$$

c) Find $V(Y_2)$. $E(Y_2^2) = \int_0^1 y_2^2 f_{Y_2}(y_2) \, dy_2 = \int_0^1 y_2^2 \left(\frac{1}{2} + y_2\right) dy_2 = \int_0^1 \frac{y_2^2}{2} + \frac{y_2^3}{3} \, dy_2$

$$= \frac{y_2^3}{6} + \frac{y_2^4}{4} \Big|_0^1 = \frac{1}{6} + \frac{1}{4} = \underline{\frac{5}{12} = 0.4167}$$

$$V(Y_2) = E(Y_2^2) - (E Y_2)^2 = \frac{5}{12} - \left(\frac{7}{12}\right)^2 = \frac{11}{144} = \boxed{0.07639}$$

d) Are Y_1 and Y_2 independent? Explain.

$\boxed{\text{NO}}$ $\underline{f(y_1, y_2) \ne g(y_1) h(y_2) \text{ on the support}}$

and/or

or $f_{Y_1}(y_1) = \frac{1}{2} + y_1, \; 0 < y_1 < 1$ by symmetry

and $f(y_1, y_2) = y_1 + y_2 \ne \left(\frac{1}{2} + y_1\right)\left(\frac{1}{2} + y_2\right) = f_{Y_1}(y_1) f_{Y_2}(y_2)$

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8) Let X be a random variable from a distribution with pdf

$$f(x) = 2x$$

for $0 < x < 1$ and $f(x) = 0$, otherwise. Let $Y = 8X^3$ and find the pdf of Y using the method of transformations. Do not forget to include the support of Y.

$y = 8x^3 = t(x)$, $t(0) = 0$, $t(1) = 8$

$x^3 = \frac{y}{8}$ or $x = \frac{1}{2} y^{\frac{1}{3}} = t^{-1}(y)$

$\left| \frac{dt^{-1}(y)}{dy} \right| = \left| \frac{1}{6} y^{\frac{1}{3}-1} \right| = \frac{1}{6} y^{-\frac{2}{3}}$

$f_Y(y) = f_X(t^{-1}(y)) \left| \frac{dt^{-1}(y)}{dy} \right| =$

$2\left(\frac{1}{2} y^{\frac{1}{3}}\right) \frac{1}{6} y^{-\frac{2}{3}} = \boxed{\frac{1}{6} y^{-\frac{1}{3}} = \frac{1}{6} \frac{1}{y^{1/3}}, \ 0 < y < 8}$

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