needs normal tables

Math 480 Exam 3, Fall 2018 Name____________________

1) Consider the following ordered data set from a Poisson(49) distribution. Find shorth(3). Q11d20

44 44 45 46 49 50 53

$45-44=1$

$46-44=2$

$49-45=4$

$50-46=4$

$53-49=4$

$\text{shorth}(3) = \boxed{[44, 45]}$

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2) Suppose $X \sim$ two parameter power($\lambda =$ 0.5, $\tau = 2$) with $F^{-1}(u) = \tau\, u^{\lambda}$. Simulate two values of x_i from this distribution if $u_1 = 0.89$ and $u_2 = 0.15$. Q11d20

$x_i = F^{-1}(u_i)$

$x_1 = 2\,(.89)^{\frac{1}{2}} = \boxed{1.8866}$

$x_2 = 2\,(.15)^{\frac{1}{2}} = \boxed{0.7746}$

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3) Suppose $X|Y \sim \text{Poisson}(Y)$ and $Y \sim \text{gamma}(\alpha = 1, \beta = \lambda = 0.5)$.

a) Find $E(X)$.

$$= E[\bar{E}(X|Y)] = E(Y) = \frac{\alpha}{\lambda} = \frac{1}{1/2} = \boxed{2}$$

8

≤ or −5

9 got it / 20

b) Find $V(X)$. $= E[V(X|Y)] + V[E(X|Y)] = E(Y) + V(Y)$

$$= \frac{\alpha}{\lambda} + \frac{\alpha}{\lambda^2} = \frac{1}{1/2} + \frac{1}{1/4} = 2 + 4 = \boxed{6}$$

7

15

4) Suppose that the number of lightning bug flashes in a backyard on July 4 is modelled by a Poisson process at a rate of 2 per minute. Calculate the probability of 0 flashes in 0.5 minutes.

$$W = N(0.5) \sim \text{poisson}[\bar{\lambda t} = 2(0.5) = 1 = \mu)$$

$$P(W=k) = \frac{e^{-\mu}\mu^k}{k!}$$

$$P(W=0) = \frac{e^{-1}\,1^0}{0!} = \boxed{e^{-1} = 0.3679}$$

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$EY_1 = \frac{\alpha}{\lambda} = \frac{2}{.01} = 200 \qquad V(Y_1) = \frac{\alpha}{\lambda^2} = \frac{2}{(.01)^2} = 20000$

5) Suppose $X(t) = \sum_{i=1}^{N(t)} Y_i$ follows a compound Poisson process where the number of claims $N(t)$ occur at a rate of $\lambda = 4$ per day and the claim severity distribution Y_i is Gamma($\alpha = 2, \lambda = 0.01$) with $200 = E(Y_i)$.

a) Find $E[X(t)]$. $= \lambda t\, EY_1 = 4t\, 200 = \boxed{800t}$

$V(X(t)) = \lambda t\, E(Y_1^2) = \lambda t[V(Y_1) + [EY_1]^2]$ for -4

4
20 Sort

b) Find $SD[X(t)]$.

$= 4t[20000 + (200)^2]$ so $SD(X(t)) = \sqrt{4(20000 + (200)^2)}\sqrt{t}$ $\underbrace{(200)^2}_{40000}$

$= \sqrt{240000}\sqrt{t} = \boxed{489.8979\sqrt{t}}$

c) Find $E[N(14)]$, the expected number of claims in a 14 day period.

$N(14) \sim \text{poisson}(\lambda t = 4(14) = 56)$

so $E[N(14)] = \boxed{56}$

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6) Suppose the number of claims per day $\{N(t), t \geq 0\}$ is a nonhomogeneous Poisson process where $\lambda(t) = 4t^3$ and t is the time in days. Find the expected number of claims $E(N(2))$ in 2 days.

$m(2) = m(2) - m(0) = \int_0^2 \lambda(t)\,dt = \int_0^2 4t^3 dt = 4\frac{t^4}{4}\Big|_0^2$

$= 2^4 = 16$

so $W = N(2) \sim \text{Poisson}(16)$

and $E(W) = E[N(2)] = \boxed{16}$

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1st transition

2nd transition

$$P = \left[\begin{array}{ccc} nonmanual & manual & farm \\ 0.594 & 0.396 & 0.010 \\ 0.211 & 0.782 & 0.007 \\ 0.252 & 0.641 & 0.107 \end{array}\right].$$

1910 1911 1912

end of 1911 or beginning of 1912

7) From a 1910 study of about 10253 workers in Marion county Indiana, a randomly selected worker person had a nonmanual, manual, or farming job with the above transition matrix (where the time period is 1 year and P works for several years). If a worker is in a manual job at the beginning of 1910, find π_2, the state vector at the end of 1911 (or beginning of 1912).

$\pi_2 = \pi_0 P P = [0\ 1\ 0] P P = [.211\ .782\ .007] P$

$.211(.594) + .782(.211) + .007(.252)$

$.211(.396) + .782(.782) + .007(.641)$

$.211(.01) + .782(.007) + .007(.107)$

so $1 - .2921 - .6996$ need to add to 1

$$= \boxed{[0.2921,\ 0.6996,\ 0.008333]}$$

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8) Suppose

$$P^{(1)} = \begin{array}{c} \begin{array}{cc} 1 & 2 \end{array} \\ \begin{bmatrix} 0.6 & 0.4 \\ 0.8 & 0.2 \end{bmatrix} \end{array} \text{ and } P^{(2)} = \begin{bmatrix} 0.7 & 0.3 \\ 0.9 & 0.1 \end{bmatrix}.$$

$\pi_2 = \begin{pmatrix} \pi_{12} \\ \pi_{22} \end{pmatrix}$

If the process begins in State 2, what is the probability that the process will be in State 1 after 2 steps?

$\pi_2 = \pi_0 P^{(1)} P^{(2)} = [0\ 1] P^{(1)} P^2 = [.8\ .2] \begin{bmatrix} .7 & .3 \\ .9 & .1 \end{bmatrix}$

$= [.8(.7) + .2(.9)\quad .8(.3) + .2(.1)]$

$= [0.74\quad .26]$ so $\boxed{0.74} = \pi_{12}$

8

9) A multiple choice final exam has 80 question. Suppose that a student guesses the answer for each of the 80 questions. Then the probability that the student correctly answers a question is 0.25. Find the approximate probability that the student answers at least 25 of the 80 questions correctly. (Hint: Let Y be the number of questions answered correctly, then Y is binomial($n = 80, p = 0.25$). Let X be a normal RV with mean $\mu = np$ and SD $\sigma = \sqrt{np(1-p)}$ and find $P(X \geq 24.5)$.) $E(X) = np = 80(.25) = 20$ Q11 d20

$SD(X) = \sqrt{80(.25)(.75)} = \sqrt{15} = 3.8730$

20 24.5

$$z = \frac{24.5 - 20}{3.8730} = 1.16$$

	.06
1.1	.8770

1.16

$\text{Prob} = 1 - .8770 = \boxed{0.1230}$

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10) Suppose a stock price $A(t)$ follows an arithmetic Brownian motion with drift $\mu = 4$ and volatility $\sigma = 3$. If $A(7) = 28$, calculate the probability that $A(9) \leq 40$. Q11 d20

10/20 got it

$W \sim A(t+\Delta) \mid A(t) \sim N(A(t) + \mu\Delta, \sigma^2\Delta)$

$t = 7, \quad \Delta = 2 \quad$ so

$W \sim N(28 + 4(2), 3^2(2)) \sim N(36, 18)$

36 40

$$z = \frac{40 - 36}{\sqrt{18}} = 0.94$$

	.04
0.9	.8264

so $P(W \leq 40) = \boxed{0.8264}$

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