state 0→1 1→2
initial ↑ ↑ ↑
transition

$= .008 + .08 + .08 + .3 = .468$

Math 480 Quiz 10 Fall 2018 Name_____________________

$$P = \begin{array}{c} \\ F \\ G \\ H \\ I \end{array} \begin{array}{c} \begin{array}{cccc} F & G & H & I \end{array} \\ \left[\begin{array}{cccc} 0.2 & 0.8 & 0.0 & 0.0 \\ 0.5 & 0.0 & 0.5 & 0.0 \\ 0.75 & 0.0 & 0.0 & 0.25 \\ 1.0 & 0.0 & 0.0 & 0.0 \end{array}\right] \end{array}.$$

0	1	2	3	
F	F	F	F	.2(.2).2
	F	G	F	.2(.8).5
	G	F	F	.8(.5).2
	G	H	F	.8(.5).75

transition made at beginning of the year

bad wording but good hint

1) A machine is in one of four states (F, G, H, I) and migrates annually among them according to a Markov chain with the above transition matrix. At time 0, the machine is in state F. Find the probability that the machine is in state F at the end of 2 years.

0→1 1→2 2→3

and makes a transition. give hint find π_3 after 3 transitions

$\pi_3 = \pi_0 P^3 = [1\ 0\ 0\ 0] P P^2 = (.2\ .8\ 0\ 0] P P$

$= [.44\ \ \underbrace{.16}_{.2(.8)}\ \ \underbrace{.4}_{.8(.5)}\ \ 0] P = (.468\ \ .352\ \ .08\ \ .1)$

$(.2)^2 + .5(.8)$

1st entry $= .44(.2) + .16(.5) + .4(.75) + 0 = \boxed{0.468}$

20

2) Suppose

$$P^{(1)} = \begin{bmatrix} 0.5 & 0.5 \\ 0.8 & 0.2 \end{bmatrix} \quad \text{and} \quad P^{(2)} = \begin{bmatrix} 0.6 & 0.4 \\ 0.7 & 0.3 \end{bmatrix}.$$

$\pi_2 = \begin{pmatrix} \pi_{12} \\ \pi_{22} \end{pmatrix}$

If the process begins in State 2, what is the probability that the process will be in State 1 after 2 steps?

$\pi_2 = \pi_0 P^{(1)} P^{(2)} = [0\ 1] \begin{bmatrix} .5 & .5 \\ .8 & .2 \end{bmatrix} P^2 =$

$[.8\ \ .2] \begin{bmatrix} .6 & .4 \\ .7 & .3 \end{bmatrix} = [.8(.6) + .2(.7)\ \ \ .8(.4) + .2(.3)]$

$= [.62\ \ .38]$ so $\boxed{0.62} = \pi_{12}$

20

3) Suppose $X(t) = \sum_{i=1}^{N(t)} Y_i$ follows a compound Poisson process where the number of claims $N(t)$ occur at a rate of $\lambda = 10$ per day and the claim severity distribution Y_i is exponential with $1500 = E(Y_i)$.

a) Find $E[X(t)]$. $= \lambda t E(Y_i) = 10 t\, 1500$ $\boxed{15000\ t}$

$V(X(t)) = \lambda t\left(V(Y_i) + (E(Y_i))^2\right) =$

b) Find $SD[X(t)]$. $10t\left[(1500)^2 + (1500)^2\right] = 20(1500)^2\ t$

So $SD[X(t)] = \sqrt{20(1500)^2}\ \sqrt{t}$ $= \sqrt{45000000}\sqrt{t}$

$= \boxed{6708.2039\ \sqrt{t}}$

c) Find $E[N(30)]$, the expected number of claims in a 30 day period.

$N(30) \sim pois(\lambda t = 10(30)) = 300$

So $E[N(30)] = \boxed{300}$

45

4) Suppose the number of claims per day $\{N(t), t \geq 0\}$ is a nonhomogeneous Poisson process where $\lambda(t) = 3t^2$ and t is the time in days. Find the probability of 7 claims in the first two days. Hint: need to find $m(2) = m(2) - m(0)$. $w = N(2) \sim Pois(m(2) = 8 = \mu)$

Since $m(2) = \int_0^2 3t^2\,dt = 3\frac{t^3}{3}\Big|_0^2 = 8$ 6pts $P(k) = \frac{e^{-\mu}\mu^k}{k!}$

So $P(N(2) = 7) = \frac{e^{-8}8^7}{7!} = \boxed{0.1396}$

$= \frac{703.5161}{5040}$

15