Math 480 Quiz 4 fall 2018 Name____________________

1) Suppose that the probability density function for a random variable Y is given by

$$f(y) = \begin{cases} c\,y, & \text{if } 0 \le y \le 4 \\ 0, & \text{otherwise.} \end{cases}$$

a) Find c.

$$\int_0^4 c y\,dy = c\frac{y^2}{2}\Big|_0^4 = 8c = 1$$

$$\boxed{c = \frac{1}{8} = 0.125}$$

b) Find $E(Y)$. $= \int_0^4 y f(y)\,dy$

$$= \int_0^4 y \frac{1}{8} y\,dy = \frac{1}{8}\int_0^4 y^2\,dy = \frac{1}{8}\frac{y^3}{3}\Big|_0^4 = \frac{64}{8(3)} = \boxed{\frac{8}{3} = 2.6667}$$

c) Find $V(Y)$. $EY^2 = \int y^2 f(y)\,dy = \int_0^4 \frac{1}{8} y^3\,dy = \frac{1}{8}\frac{y^4}{4}\Big|_0^4$

$$= \frac{256}{8(4)} = \frac{32}{4} = \underline{8}$$

$$V(Y) = EY^2 - (EY)^2 = 8 - \left(\frac{8}{3}\right)^2 = \frac{72-64}{9} = \boxed{\frac{8}{9} = 0.8889}$$

d) Find $F(y)$.

$$= \int_0^y f(t)\,dt = \int_0^y \frac{1}{8} t\,dt = \frac{1}{8}\frac{t^2}{2}\Big|_0^y = \begin{cases} 0 & y < 0 \\ \frac{y^2}{16} & 0 \le y \le 4 \\ 1 & y > 4 \end{cases}$$

check $\frac{d}{dy}\frac{y^2}{16} = \frac{y}{8}$

or -1

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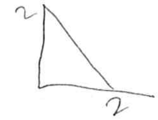
14 each



area $= \frac{1}{2}(2)(4) = 4$

so $c = \frac{1}{2}$

2) Suppose that the probability density function for a random variable Y is given by

$$f(y) = \begin{cases} c\,(2-y), & \text{if } 0 \le y \le 2 \\ 0, & \text{otherwise.} \end{cases}$$

a) Find c. $1 = c\int_0^2 (2-y)\,dy = c\left(2y - \frac{y^2}{2}\Big|_0^2\right) = c(4-2) = 2c$

$\boxed{\text{so } c = \frac{1}{2} = 0.5}$

b) Find $E(Y)$. $= \int y f(y)\,dy = = \int_0^2 y\,\frac{1}{2}(2-y)\,dy = \int_0^2 \frac{1}{2}(2y - y^2)\,dy$

$= \frac{1}{2}\left(\frac{2y^2}{2} - \frac{y^3}{3}\Big|_0^2\right) = \frac{1}{2}\left(4 - \frac{8}{3}\right) = \boxed{\frac{4}{6} = \frac{2}{3} = 0.6667}$

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10/20 got it

3) Suppose

$$F(x) = 1 - \exp\left[\frac{-(e^x - 1)}{\lambda}\right]$$

for $x > 0$ where $\lambda > 0$. Find the pdf $f(x)$ for $x > 0$.

≈ 4

$= F'(x) = -\exp\left(-\frac{(e^x - 1)}{\lambda}\right)\left(\frac{-e^x}{\lambda}\right)$

$= \boxed{\frac{e^x}{\lambda}\exp\left[\frac{-(e^x-1)}{\lambda}\right]}$

$= \frac{1}{\lambda}\exp\left(x - \frac{e^x - 1}{\lambda}\right)$

$= \frac{e^x}{\lambda}\exp\left(\frac{1-e^x}{\lambda}\right)$

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