Math 480 Quiz 5 fall 2018 Name_____

1) Suppose that the joint probability function $p(y_1, y_2)$ of Y_1 and Y_2 is and is tabled as shown. Find the marginal probability function $p_{Y_1}(y_1)$ for Y_1.

$p(y_1, y_2)$		y_2 1	2	3	4	
y_1	10	0.1	0.2	0.1	0.05	.45
	15	0.05	0.05	0.07	0.03	.20
	20	0.08	0.10	0.15	0.02	.35

y_1	10	15	20
$p_{Y_1}(y_1)$	.45	.20	.35

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2) The IQ test scores X used for admission by MENSA are normally distributed with mean $\mu = 100$ and standard deviation $\sigma = 16$.

a) What is the score such that 2% of scores are higher?

.98 X .02

.05 .06

2.0 | .9798 .9803

$z^* = 2.05$

$x^* = \mu + \sigma z^* = 100 + 16(2.05) = 132.8$

b) Find the probability that X will be less than 92.

92 100

$\frac{92-100}{16} = -0.50$

-0.5 | .00 | .3085

$P(X < 92) = P(Z < -0.50) = 0.3085$

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3) Suppose that the spring ACT exam is standardized to have mean $\mu = 22$ and standard deviation $\sigma = 3$. You may assume that the scores follow a normal curve.

a) About what proportion of students will have scores between 22 and 28?

$z = 0, \quad z = \frac{28-22}{3} = 2.0$

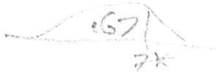
	.00
0.0	.5
2.0	.9772

$P(22 < x < 28) = P(0 < z < 2.0) = .9772 - .5 = \boxed{0.4772}$

b) What ACT score is such that 67% of all ACT scores are worse?

	.04
.40	.6700

$z^* = .44$



$x^* = \mu + \sigma z^* = 22 + 3(.44) = \boxed{23.22}$

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4) Suppose that the joint pdf of the random variables Y_1 and Y_2 is given by

$$f(y_1, y_2) = \begin{cases} c\, y_1^2 y_2, & \text{if } 0 \le y_1 \le 1, 0 \le y_2 \le 3 \\ 0, & \text{otherwise.} \end{cases} \quad = c \int_0^1 y_1^2 dy_1 \int_0^3 y_2 dy_2$$

a) Find c. $1 = c \int_0^3 \int_0^1 y_1^2 y_2 \, dy_1 y_2 = c \int_0^3 \left[\frac{y_1^3}{3}\right]_0^1 \left[\frac{y_2}{2}\right] dy_2 = c \int_0^3 \frac{y_2}{3} dy_2$

$= c \frac{y_2^2}{6} \Big|_0^3 = c\frac{9}{6} = 1$ or $c = \boxed{\frac{6}{9} = \frac{2}{3} = 0.6667}$

or $1 = \int_0^1 \int_0^3 y_1^2 y_2 \, dy_2 \, dy_1 = c \int_0^1 y_1^2 \left[\frac{y_2^2}{2}\right]_0^3 dy_1 = c \int_0^1 \frac{9}{2} y_1^2 dy_1$

$= c \frac{9}{2} \frac{y_1^3}{3} \Big|_0^1 = c\frac{3}{2} = 1 \quad$ or $c = \frac{2}{3}$

b) Find the marginal pdf of Y_1. Include the support.

$f_{Y_1}(y_1) = \int_0^3 f(y_1, y_2) dy_2 = \int_0^3 \frac{2}{3} y_1^2 y_2 dy_2 = \frac{2}{3} y_1^2 \frac{y_2^2}{2} \Big|_0^3$

$= 3 y_1^2$

so $\boxed{f_{Y_1}(y_1) = 3 y_1^2, \quad 0 \le y_1 \le 1}$

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