Math 480 Quiz 6 fall 2018 Name____________________

1) Suppose that the joint pmf of Y_1 and Y_2 is given by the table below.

$p(y_1, y_2)$		y_2: 0	1	
	0	0	1/2	1/2
y_1	1	1/2	0	1/2
		1/2	1/2	

a) Are Y_1 and Y_2 independent? Explain.

(no) Support is not a cross product

(or $p(0,0) = 0 \neq p_{Y_1}(0)\, p_{Y_2}(0) = \frac{1}{2}\frac{1}{2} = \frac{1}{4}$)

b) Find the marginal probability ~~function~~ mass function $p_{Y_2}(y_2)$ for Y_2.

y_2	0	1
$p_{Y_2}(y_2)$	$\frac{1}{2}$	$\frac{1}{2}$

c) $E(Y_1) = E(Y_2) = 0.5$. Find $\text{Cov}(Y_1, Y_2)$. $E\,Y_1Y_2 = 0(0)0 + 0(1)\frac{1}{2} + 0(1)\frac{1}{2} + 1(1)0 = 0$

so $\text{Cov}(Y_1,Y_2) = E\,Y_1Y_2 - EY_1\,EY_2 = 0 - \frac{1}{2}\frac{1}{2}$

$= \boxed{-\frac{1}{4} = -0.25}$

39

2) Suppose that the moment generating function (mgf) of a random variable Y is

$$\phi(t) = \frac{\log(1 - \theta e^t)}{\log(1 - \theta)}$$

for $t < -\log(\theta)$ where $\theta > 0$ is a **known constant**. Using the mgf $\phi(t)$, find $\phi'(t)$ and $E(Y)$. **Note that the denominator is a constant.**

$\phi'(t) = \frac{1}{\log(1-\theta)} \frac{1}{1-\theta e^t}(-\theta e^t)$, so $E(Y) = \phi'(0) =$

log rule

$$\boxed{\frac{-1}{\log(1-\theta)}\frac{\theta}{1-\theta} = \frac{-\theta}{(1-\theta)\log(1-\theta)} = \frac{\theta}{(\theta-1)\log(1-\theta)}}$$

12

Suppose that the joint pdf of the random variables Y_1 and Y_2 is given by

$$f(y_1, y_2) = 6y_1^2 y_2$$

if $0 < y_1 < 1$ and $0 < y_2 < 1$, and $f(y_1, y_2) = 0$, otherwise.

a) Are Y_1 and Y_2 independent? Explain.

$\boxed{\text{Yes}}$ $f(y_1, y_2) = g(y_1)h(y_2) = \underbrace{3y_1^2}\,\underbrace{(2y_2)} = (6y_1^2)\,y_2$

12

on cross product support $f_{Y_1}(y_1)\, f_{Y_2}(y_2)$

b) Find the marginal pdf of Y_1. Include the support.

$\int_{-\infty}^{\infty} f(y_1, y_2)\,dy_2 = \int_0^1 6y_1^2\, y_2\, dy_2 = 6y_1^2 \frac{y_2^2}{2}\Big|_0^1$

$= \boxed{3y_1^2, \quad 0 < y_1 < 1}$

12

c) Find $E(Y_1)$. $= \int y_1 f(y_1)\,dy_1 = \int_0^1 3y_1^3\, dy_1 = \frac{3y_1^4}{4}\Big|_0^1$

$= \boxed{\frac{3}{4} = 0.75}$

12

d) Find $\operatorname{Cov}(Y_1, Y_2)$. $= 0$ by ind

13

hard way

$EY_1Y_2 = \frac{1}{2} \overset{\text{ind}}{=} EY_1\,EY_2$

$EY_2 = \frac{2}{3}$

$\operatorname{cov}(Y_1, Y_2) = \frac{1}{2} - \frac{3}{4}\cdot\frac{2}{3} = 0$

49