Math 480 Quiz 7 fall 2018 Name____________________

x	-3	-1	1	2
f(x) $P_X(x)$	3/20	7/20	6/20	4/20
$Y=x^2+x$	6	0	2	6

1) Let the discrete random variable X have a probability mass function given by the table above. a) Find the pmf of $Y = X^2 + X$.

y	0	2	6
$P_Y(y)$	$\frac{7}{20}=.35$	$\frac{6}{20}=.3$	$\frac{7}{20}=.35$

b) Find the moment generating function mgf of Y.

$$= Ee^{tY} = \sum e^{ty} P_Y(y) = \boxed{\frac{7}{20} + e^{t2}\frac{6}{20} + e^{t6}\frac{7}{20}}$$

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2) Suppose that X is a random variable with pdf

$$f(x) = 3x^2 \quad \text{where} \quad 0 < x < 1.$$

$\frac{1}{20}$ got it

Let $Y = X^{-1/2}$ and find the pdf of Y. Do not forget to include the support of Y.

$Y = \frac{1}{\sqrt{x}} = x^{-\frac{1}{2}} = t(x)$, $\quad t(1) = 1 \quad t(0) = \lim_{x \downarrow 0} \frac{1}{\sqrt{x}} = \infty$

$y^2 = \frac{1}{x}$, $x = \frac{1}{y^2} = y^{-2} = t^{-1}(y)$, $\left|\frac{dt^{-1}(y)}{dy}\right| = |-2y^{-3}| = 2y^{-3} = \frac{2}{y^3}$

$$f_Y(y) = f_X(t^{-1}(y)) \left|\frac{dt^{-1}(y)}{dy}\right| = \underbrace{3\left(\frac{1}{y^2}\right)^2 \frac{2}{y^3}}_{-5} = \frac{6}{y^4 y^3} = \frac{6}{y^7}$$

$$\boxed{f_Y(y) = \frac{6}{y^7} = 6y^{-7}, \quad y > 1}$$

or -4

$f_Y(y) < 0$ at least -6

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Check $\int_1^\infty \frac{6}{y^7}\,dy = \int_1^\infty 6y^{-7}\,dy = 6\frac{y^{-6}}{-6}\Big|_1^\infty = 0 - (-1) = 1$

3) Suppose that the joint probability mass function $p(y_1, y_2)$ of Y_1 and Y_2 is and is tabled as shown.

$p(y_1, y_2)$		y_2 = 1	2	3	4	$P_{Y_1}(y_1)$
y_1	1	0.062	0.192	0.176	0.210	.640
	2	0.006	0.011	0.008	0.006	.031
	3	0.083	0.115	0.070	0.061	.329

a) Find the marginal probability mass function $p_{Y_1}(y_1)$ for Y_1.

y_1	1	2	3
$P_{Y_1}(y_1)$	.640	.031	.329

b) Find the conditional probability mass function $p(y_2|y_1)$ of Y_2 given $Y_1 = 2$. $P_{Y_2|Y_1=2}(y_2|2) = \frac{P(2, y_2)}{P_{Y_1}(2)}$

$\frac{.006}{.031}$ $\frac{.011}{.031}$ $\frac{.008}{.031}$ $\frac{.006}{.031}$ ← fix $Y_1 = 2$ on table

y_2	1	2	3	4
$P_{Y_2\|Y_1=2}(y_2\|2)$	.1935	.3548	.2581	.1935

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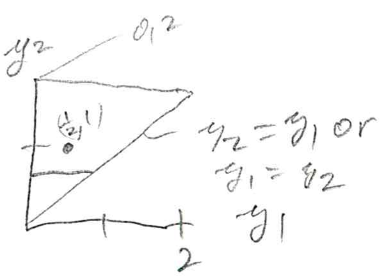
4) Suppose that the joint pdf of the random variables Y_1 and Y_2 is given by

$$f(y_1, y_2) = \begin{cases} \frac{1}{2} & \text{if } 0 \le y_1 \le y_2 \le 2 \\ 0, & \text{otherwise.} \end{cases}$$



Suppose that the marginal pdf of Y_2 is given by

$$f_{Y_2}(y_2) = \begin{cases} \frac{y_2}{2} & \text{if } 0 \le y_2 \le 2 \\ 0, & \text{otherwise.} \end{cases}$$

Find the conditional pdf of Y_1 given $Y_2 = y_2$, that is, find $f_{Y_1|Y_2=y_2}(y_1|y_2) \equiv f(y_1|y_2)$. Make sure you include the support of the conditional pdf. (Hint: sketch the support and draw in a horizontal line corresponding to a fixed value of y_2.)

$$f_{Y_1|Y_2=y_2}(y_1|y_2) = \frac{f(y_1, y_2)}{f_{Y_2}(y_2)} = \frac{\frac{1}{2}}{\frac{y_2}{2}} = \boxed{\frac{1}{y_2}, \quad 0 < y_1 < y_2}$$

−5 no sketch
−4 sketch

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