Math 480 Quiz 8 fall 2018 Name____________________

1) Suppose Y is the number of gypsy moths caught in a trap for the moth. Assume that Y is from a highly skewed distribution with mean $\mu = 0.5$ and standard deviation $\sigma = 0.7$. Assume that the sample mean $\overline{Y}$ is computed from a sample of size $n = 16$ traps. Find $P(\overline{Y} \geq 0.6)$, if possible.

not possible

$\overline{20}$

2) Suppose Y is the number of gypsy moths caught in a trap for the moth. Assume that Y is from an approximately normal distribution with mean $\mu = 0.5$ and standard deviation $\sigma = 0.7$. Assume that the sample mean $\overline{Y}$ is computed from a sample of size $n = 16$ traps. Find $P(\overline{Y} \geq 0.6)$, if possible.

$\mu_{\bar{Y}} = \mu = 0.5$, $\sigma_{\bar{Y}} = \frac{\sigma}{\sqrt{n}} = \frac{0.7}{\sqrt{16}} = 0.175$

$z = \frac{.6 - .5}{.175} = \frac{.6 - .5}{0.7/\sqrt{16}} = 0.57$ .7157

$P(\bar{Y} \geq 0.6) = P(Z \geq 0.57) = 1 - P(Z \leq 0.57) = 1 - .7157 = \boxed{0.2843}$

$\overline{20}$

3) Hurricanes occur at a Poisson rate of 0.25 per week during hurricane season which lasts 15 weeks. Find the probability of four hurricanes in the upcoming hurricane season.

$W \sim \text{Poisson}\left(\mu = \lambda t = 0.25(15) = \frac{15}{4} = 3.75\right)$

$P(W = k) = p(k) = \frac{e^{-\mu} \mu^k}{k!}$

$p(4) = \frac{e^{-3.75} (3.75)^4}{4!} = \boxed{0.1938}$

$\overline{20}$

4) Suppose that $X_1, \ldots, X_n$ are independent random variables where $E(X_i) = 4p$, $V(X_i) = 8p$ and the moment generating function of X_i is $\phi_{X_i}(t) = (1 - 2t)^{-2p}$ for $t < 0.5$. Let $U = \sum_{i=1}^{n} X_i$.

a) Find E(U).

b) Find the variance V(U) of U.

c) Find the moment generating function $\phi_U(t)$ of U.

5) Let X be a random variable from a distribution with pdf

$$f(x) = \frac{\theta}{x^2} \exp\left(\frac{-\theta}{x}\right)$$

where $x > 0$ and $\theta > 0$. Let $Y = \frac{1}{X}$ and find the pdf of Y using the method of transformations. Do not forget to include the support of Y.