last 2 pages of E2 rev

Math 480 Quiz 9 Fall 2018 Name___________________

YOU ARE BEING GRADED FOR WORK

1) Suppose $X|Y \sim \text{binomial}(Y, p_1)$ and $Y \sim \text{binomial}(n, p_2)$.

a) Find $E(X)$. $= E(E(X|Y)) = E[Y p_1] = p_1 E(Y) = \boxed{n p_1 p_2}$

b) Find $V(X)$. $= E[V(X|Y)] + V[E(X|Y)] =$

$E[Y p_1(1-p_1)] + V(Y p_1) = p_1(1-p_1)E(Y) + p_1^2 V(Y)$

$= p_1(1-p_1) n p_2 + p_1^2 n p_2 (1-p_2)$

$= n p_1 p_2 (1-p_1) + p_1^2 n p_2 (1-p_2)$

$= n p_1 p_2 [1-p_1 + p_1(1-p_2)] = n p_1 p_2 (1-p_1 p_2)$

$\overline{28}$

2) Suppose a Math department hires lecturers in a Poisson process at a rate of 3 per year. Calculate the probability that at least 2 lecturers are hired during the first six months. (0.5 years)

$W = N(0.5) \sim \text{Poisson}(3(0.5) = 1.5 = \mu)$, $P(W=k) = \dfrac{e^{-\mu}\mu^k}{k!}$

$P(W \geq 2) = 1 - P(W < 2) = 1 - P(W=0) - P(W=1)$ or

$= 1 - \dfrac{e^{-1.5}(1.5)^0}{0!} - \dfrac{e^{-1.5}(1.5)^1}{1!} =$

$1 - 0.2231 - 0.3347 = 1 - 0.5578 =$

$\overline{15}$

$\boxed{0.4422}$

3) Suppose $X(t) = \sum_{i=1}^{N(t)} Y_i$ follows a compound Poisson process where the number of claims $N(t)$ occur at a rate of $\lambda = 9$ per day and the claim severity distribution Y_i is exponential with $\theta = 500$. $= EY_i$

a) Find $E[X(t)]$.

$= \lambda t \, EY_1 = 9t \; 500 = \boxed{4500t}$

14

b) Find $SD[X(t)]$.

$V(X(t)) = \lambda t \, E(Y_1^2) = \lambda t \left(V(Y_1) + [E(Y_1)]^2\right)$

$= 9t \, [(500)^2 + (500)^2] = 18(500)^2 t$

So $SD(X(t)) = \sqrt{V(X(t))} = \sqrt{18(500)^2}\sqrt{t} = \boxed{2121.3203\sqrt{t}}$

14

c) Find $E[N(7)]$, the expected number of claims in a 7 day period.

$W = N(7) \sim \text{Poisson}[\lambda t = 9(7) = 63]$

So $E(W) = E[N(7)] = \boxed{63}$

14

42

4) Suppose the number of claims per day $\{N(t), t \geq 0\}$ is a nonhomogeneous Poisson process where $\lambda(t) = 2t$ and t is the time in days. Find the expected number of claims $E(N(1.2))$ in 1.2 days.

$m(1.2) = m(1.2) - m(0) = \int_0^{1.2} \lambda(t)\,dt = \int_0^{1.2} 2t\,dt = \frac{2t^2}{2}\Big|_0^{1.2}$

$= (1.2)^2 = 1.44.$

So $W = N(1.2) \sim \text{Poisson}(1.44)$

and $E(W) = E[N(1.2)] = \boxed{1.44}$

15