ii) $X(t)$ = aggregate ... (t) ... amount

over $(r, r+t]$. ${}_hX_r = X(r+h) - X(r)$ = aggregate loss over $(r, r+h]$ has a compound Poisson dist with parameters λh and $\underbrace{E(Y_1) \text{ and } E(Y_1^2)}_{F_Y(y)}$.

ex) Let $N(t)$ be a Poisson process with mean 100 claims per month. 2% of these claims exceed 30000. Want to count these claims with $N_S(t)$. Then $N_S(t)$ is a Poisson process with rate $\lambda_S = 0.02(100) = 2$ per month (eg deductible = 30000).

ex) Suppose $N(t)$ is a nonhomogeneous Poisson process with
$$\lambda(t) = \begin{cases} 5+5t, & 0 \le t \le 3 \\ 20, & 3 \le t \le 5 \\ 20 - 2(t-5), & 5 \le t \le 9 \end{cases}$$
that counts $\underbrace{\#}_{\text{number}}$ of arrivals. Find the expected # of arrivals for time between $\frac{1}{2}$ and $\frac{3}{2}$.

sol'n) Let W count the #.
$W \sim \text{Poisson}\left(m\left(\frac{3}{2}\right) - m\left(\frac{1}{2}\right)\right)$. So answer $= E(W)$

$$\int_{\frac{1}{2}} (5+5x)\,dx = 5x + \frac{5x^2}{2}\Big|_{1/2}^{3/2} =$$

$$5\frac{3}{2} + \frac{5}{2}\frac{9}{4} - \frac{5}{2} - \frac{5}{2}\frac{1}{4} = \frac{60+45-20-5}{8} = \frac{80}{8} = \boxed{10}$$

ex) $EY_1 = \frac{5}{2}$, $EY_i^2 = \frac{43}{6}$ and $N(t)$ is a Poisson process with rate $\lambda = 2$ per week,

Y_i = # of people in family $\in \{1,2,3,4\}$, want the expected value and variance of the total # of people in 5 weeks.

Soln) $X(t) = \sum_{i=1}^{N(t)} Y_i$ is a compound process

with $\lambda t = 2(5)$,

so $E[X(t)] = \lambda t\, E(Y_1) = 2(5)\frac{5}{2} = \boxed{25}$

$V[X(t)] = \lambda t\, E(Y_i^2) = 2(5)\frac{43}{6} = \frac{430}{6} = \boxed{71.666}$

ex) Y_n = life insurance claim paid to beneficiary at time of policy holder's death. If deaths follow a Poisson process $N(t)$ with rate λ and the Y_i are iid, then $X(t) = \sum_{i=1}^{N(t)} Y_i$ is a compound Poisson process.

Then

$X(t)$ is the RV for the total amount ... of claims the insurance company will have to pay from time 0 to t. $X(t)$ is important so the insurance company knows how large a reserve should be on hand to pay the claims.

32} Let $\phi_Y(r)$ be the mgf of Y_1. Let $\phi_{X(t)}(r)$ be the mgf of RV $X(t)$. Then ← fixed

$$\phi_{X(t)}(r) = \exp\left[\lambda t\left(\phi_Y(r) - 1\right)\right].$$

p280

33} Back to the poisson process: Stationary and independent increments is basically asserting that at any point of time, the process probabalistically restarts itself or the process has no memory. So exponential interarrival times are reasonable since the Exp dist has the <u>memoryless property</u>: if $X \sim Exp(\lambda)$

$$P(X > t+s \mid X > s) = P(X > t) \quad \forall\, t, s > 0.$$

occur at a poisson rate of 0.25 (92.5)
per week during hurricane season which lasts 15 weeks. Find the prob of at least 3 and no more than 5 hurricanes during hurricane season.

Soln: Let W count # hurricanes during hurricane season. Then W ~ Poisson ($\lambda t = .25(15) = \frac{15}{4} = 3.75 = \mu$). So W ~ Poisson ($\mu = 3.75$)

and $P(W=k) = \dfrac{e^{-\mu}\mu^k}{k!} = P(k)$. want

$P(3) + P(4) + P(5) =$

$$\frac{e^{-3.75}(3.75)^3}{3!} + \frac{e^{-3.75}(3.75)^4}{4!} + \frac{e^{-3.75}(3.75)^5}{5!}$$

$= e^{-3.75}(23.2086) = 0.2067 + 0.1938 + 0.1453$

$= \boxed{0.5458}$

Note that the expected number of hurricanes during hurricane season is 3.75

ex) Suppose the number of claims $\{N(t), t \geq 0\}$ is a nonhomogeneous Poisson process where $\lambda(t)$, the intensity function (claims per day) varies with t, the time in days.

$$\lambda(t) = \begin{cases} 1 & 0 \leq t < 1 \\ 2 & 1 \leq t < 2 \\ 3 & 2 \leq t. \end{cases}$$

Find $E[N(3)]$.

soln $W = N(3) \sim \text{Poisson}(m(3)) \sim \text{Poisson}(m(3) - \overset{0}{m(0)})$

$m(3) = m(3) - m(0) = \int_0^3 \lambda(t)\,dt =$

$\int_0^1 1\,dt + \int_1^2 2\,dt + \int_2^3 3\,dt = t\Big|_0^1 + 2t\Big|_1^2 + 3t\Big|_2^3$

$= (1-0) + 2(2-1) + 3(3-2) = 6.$

so $W = N(3) \sim \text{Poisson}\{m(3) = 6\}$

and $E(W) = E(N(3)) = \boxed{6}$

ex) Suppose $X(t) = \sum_{i=1}^{N(t)} Y_i$ follows a compound Poisson process where the number of claims $N(t)$ occur at a rate of $\lambda = 10$ per day and the claim severity distribution Y_i is exponential with $\theta = \frac{1}{5000}$ (or mean $= 5000$).

a) Find $E[X(t)]$.

$= \lambda t\, E(Y_1) = 10t\, 5000 = \boxed{50000t}$

S3.5

b) Find the standard deviation $SD[X(t)]$.

$V[X(t)] = \lambda t\, E(Y_1^2) = \lambda t\,[V(Y_1) + E(Y_1)^2]$

$= 10t \left[\frac{1}{\left(\frac{1}{5000}\right)^2} + \left(\frac{1}{\frac{1}{5000}}\right)^2 \right] = 20(5000)^2 t$

So $SD(X(t)) = \sqrt{V(X(t))} = \sqrt{20(5000)^2}\,\sqrt{t} =$

$\boxed{22360.6798\sqrt{t}}$

c) Find the standard deviation $SD[X(365)]$

$= \sqrt{20(5000)^2\, 365} = \boxed{427200.1873}$

d) Find $E[N(7)]$, the expected number of claims in a 7 day period.

Soln) $W = N(7) \sim \text{Poisson}[\mu = \lambda t = 10(7) = 70]$

$E[N(7)] = E(W) = \boxed{70} = \lambda t = 10(7)$

ex) Suppose the number of claims per day $\{N(t), t \geq 0\}$ is a nonhomogeneous Poisson process where $\lambda(t) = 2t$ and t is the time in days

a) Find the expected number of claims $E[N(4)]$ in the 1st 4 days.

Soln) $W = N(4) \sim \text{Poisson}[m(4) = m(4) - m(0)]$

$$m(4) = m(4) - m(0) = \int_0^4 \lambda(t)\,dt = \int_0^4 2t\,dt = \frac{2t^2}{2}\Big|_0^4 = 16$$

So $E(W) = E[N(4)] = m(4) = \boxed{16}$

b) Find the probability of at least 15 and no more than 17 claims in the 1st 4 days.

Soln $W \sim \text{Poisson}(\mu = 16)$ counts the # of claims in the 1st 4 days by a),

Let $P(k) = P(W = k) = \dfrac{e^{-\mu}\mu^k}{k!}$, want

$$P(15) + P(16) + P(17) = \frac{e^{-16}(16)^{15}}{15!} + \frac{e^{-16}(16)^{16}}{16!} + \frac{e^{-16}(16)^{17}}{17!}$$

$$= 0.09922 + 0.09922 + 0.09334 = \boxed{0.2918}$$

skip 5.5

First review matrix multiplication (5.4.5)

1) A matrix is a rectangular array of numbers. (Tend to use capital letters like A, B)

eg $\begin{bmatrix} 4 & 3 \end{bmatrix}$, $\begin{bmatrix} 3 \\ 4 \end{bmatrix}$, $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$.

1×2 $\quad 2 \times 1$ $\quad 2 \times 2$ square $\quad 2 \times 3$

2) If a matrix A has r rows and c columns, then the dimension of A is $r \times c$.

3) If $r = c = k$ then A is a <u>square matrix</u>.

4) notation

$$\begin{array}{c c} & \begin{array}{cccc} \text{column } 1 & \text{col } 2 & & \text{col } c \end{array} \\ \begin{array}{c} \text{row } 1 \\ \text{row } 2 \\ \vdots \\ \text{row } r \end{array} & A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1c} \\ a_{21} & a_{22} & \cdots & a_{2c} \\ \vdots & & & \\ a_{r1} & a_{r2} & \cdots & a_{rc} \end{bmatrix} = [a_{ij}] \end{array}$$

for $i = 1, \ldots, r$ and $j = 1, \ldots, c$. Here a_{ij} is the ij th entry (element) of A that is in the ith row and jth column.

ex) $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $\quad a_{21} = 3$, $a_{12} = 2$

5) A <u>row vector</u> is a $1 \times c$ matrix.

ex $\begin{bmatrix} 1 & 2 & 3 & 4 \end{bmatrix}$

ex. $\begin{bmatrix}1\\2\\3\\4\end{bmatrix} = \underset{4\times1}{\underline{x}}$.

The transpose of a column vector $\underset{r\times1}{\underline{x}}$ is $\underset{1\times r}{\underline{x}^T}$

$\underline{x}^T = [1\ \ 2\ \ 3\ \ 4]$

7} Let $\underline{x} = \begin{pmatrix}x_1\\ \vdots\\ x_r\end{pmatrix}$ and $\underline{y} = \begin{pmatrix}y_1\\ \vdots\\ y_r\end{pmatrix}$,

Then $\underline{x}^T\underline{y} = \underline{y}^T\underline{x}$ = vector product

$= \sum_{i=1}^{r} x_i y_i = x_1 y_1 + x_2 y_2 + \cdots + x_r y_r$.

8} Let $\underset{m\times r}{A} = [a_{ij}] \equiv \begin{bmatrix}\underline{a}_1^T\\ \vdots\\ \underline{a}_m^T\end{bmatrix}$ and $\underset{r\times n}{B} = [b_{ij}] = [\underline{b}_1 \cdots \underline{b}_n]$.

The product $\underset{m\times r}{A}\,\underset{r\times m}{B} = \underset{m\times n}{D} = [d_{ij}]$ where $d_{ij} = \underline{a}_i^T\underline{b}_j$ is the vector product of the ith row of A with the jth column of B. So $d_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + \cdots + a_{ir}b_{rj}$ $= \sum_{k=1}^{r} a_{ik}b_{kj}$. Matrix multiplication is possible iff the number of columns of the matrix on the left is equal to the number of rows of the matrix on the right.

\#columns \#rows

$\underset{m\times r}{A}\ \underset{r\times n}{B} \Rightarrow \underset{m\times n}{D}$ ← dimension

9} i) In gener... $AB \neq BA$.

ii) $ABC = A(BC) = (AB)C$ if A is $m \times r$, B is $r \times p$ and C is $p \times n$. 55.5

ex} Find $AB = D = [d_{ij}]$

$$AB = \begin{bmatrix} -2 & 1 & 3 \\ 4 & 1 & 6 \end{bmatrix} \begin{bmatrix} 3 & -2 \\ 2 & 4 \\ 1 & -3 \end{bmatrix} = \begin{bmatrix} -1 & -1 \\ 20 & -22 \end{bmatrix}$$

$$d_{11} = \begin{bmatrix} -2 & 1 & 3 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} = -2(3) + 1(2) + 3(1) = -1$$

$$d_{12} = \begin{bmatrix} -2 & 1 & 3 \end{bmatrix} \begin{bmatrix} -2 \\ 4 \\ -3 \end{bmatrix} = -2(-2) + 1(4) + 3(-3) = -1$$

$$d_{21} = \begin{bmatrix} 4 & 1 & 6 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} = 4(3) + 1(2) + 6(1) = 20$$

$$d_{22} = \begin{bmatrix} 4 & 1 & 6 \end{bmatrix} \begin{bmatrix} -2 \\ 4 \\ -3 \end{bmatrix} = 4(-2) + 1(4) + 6(-3) = -22$$

with practice, omit this work

You can show $BA = \begin{bmatrix} -14 & 1 & -3 \\ 12 & 6 & 30 \\ -14 & -2 & -15 \end{bmatrix}$.

Hence $\underset{2\times 2}{AB} \neq \underset{3\times 3}{BA}$.

$\underset{1\times k}{\pi}$ $\underset{k\times k}{IP}$ row vector times square matrix

10} p183-184 A finite <u>Markov Chain</u> $\{X_n : n = 0, 1, 2, \ldots\}$ is a discrete stochastic process for which time only takes on integer values. X_n will have J possible values where often J = 2, 3 or 4. If $X_n = i \in \{1, \ldots, J\}$ then <u>the Markov Chain is in state i at time n.</u>