ex3 Suppose Y counts the number of radioactive particles that hit a target in 2 minutes and that Y is Poisson with $E(Y) = 6$.

a) Find λ

b) Find $V(Y)$

c) Find $P(Y=1)$

d) Find $P(Y \le 1)$

soln a) $\lambda = E(Y) = 6$

b) $V(Y) = E(Y) = 6$

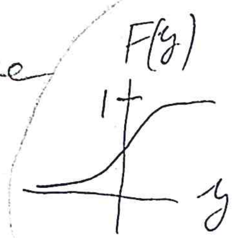
$$p(y) = \frac{\lambda^y e^{-\lambda}}{y!}$$

c) $p(1) = P(Y=1) = \frac{6^1 e^{-6}}{1!} = 6e^{-6} = \boxed{0.01487}$

d) $p(0) + p(1) = \frac{6^0 e^{-6}}{0!} + .01487 = e^{-6} + 0.01487$

$= \boxed{0.01735}$

32) Know The RV Y is continuous if the cdf $F(y)$ is continuous for $-\infty < y < \infty$



33) Know p30 Let $F(y)$ be the cdf of continuous Y and suppose $f(y) = \frac{dF(y)}{dy}$ wherever

... then $f(y)$ is called the

Probability density function (pdf) of Y. (19.5)

(Assume $\frac{dF}{dy}$ exists and is continuous except at most a finite # of points in any finite interval.)

34) Know $F(y) = \int_{-\infty}^{y} f(t)\,dt$.

35) If $f(y)$ is a pdf then

pdf 1) $f(y) \geq 0 \quad \forall\, y \in (-\infty, \infty)$

2) $\int_{-\infty}^{\infty} f(y)\,dy = 1$

36) If Y has pdf $f(y)$ and $a \leq b$, then

$$P(a \leq Y \leq b) = F(b) - F(a) = \int_a^b f(y)\,dy.$$

37) For a discrete RV W,

$P(a < W \leq b) = F(b) - F(a)$ since $F(a) = P(W \leq a)$.

If Y is a continuous RV, then $F(a) = P(Y \leq a)$

$= P(Y < a)$ since $P(Y = a) = 0$.

38) Common E1 problem: a) Find $f(y)$ from $F(y)$.

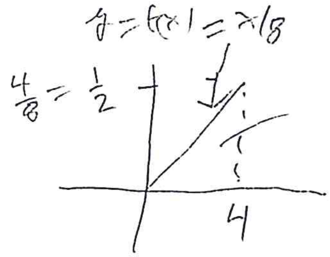
b) Find $F(y)$ from $f(y)$.

c) Told $f(y) = c\,g(y)$. Find c. see HW4, Q4.

ex) X RV with $f(x) = \begin{cases} cx & 0<x<4 \\ 0 & \text{else} \end{cases}$

$\int_{-\infty}^{\infty} f(x)\,dx = 1$ so $1 = \int_0^4 cx\,dx = c\frac{x^2}{2}\Big|_0^4 = 8c = 1$

or $c = \frac{1}{8}$. So $f(x) = \begin{cases} x/8 & 0<x<4 \\ 0 & \text{else} \end{cases}$



area of triangle $= 1 = \frac{1}{2}\left(\frac{1}{2}\right)4$

Now $F(x) = \begin{cases} 0 & x \le 0 \\ 1 & x \ge 4 \\ \frac{x^2}{16} & 0<x<4 \end{cases}$

$\int_0^x \frac{t}{8}\,dt = \frac{t^2}{2(8)}\Big|_0^x =$

check: Note that $\frac{d}{dx}F(x) = \begin{cases} 0, & x \le 0 \\ 0, & x \ge 4 \\ \frac{2x}{16} = \frac{x}{8}, & 0<x<4. \end{cases}$

$P(1 \le X \le 2) = \int_1^2 \frac{x}{8}\,dx = \frac{x^2}{16}\Big|_1^2 = \frac{4}{16} - \frac{1}{16} = \boxed{\frac{3}{16}} = F(2) - F$

$P(X \le 1 \mid X \le 2) = \frac{P(\{X \le 1\} \cap \{X \le 2\})}{P(X \le 2)} = \frac{P(X \le 1)}{P(X \le 2)} =$

$\frac{F(1)}{F(2)} = \frac{1/16}{4/16} = \boxed{\frac{1}{4}}$

If Y has pdf $f(y)$, then the expected value of Y is $E(Y) = \int_{-\infty}^{\infty} y\, f(y)\,dy$ and

20.5

$E[g(Y)] = \int_{-\infty}^{\infty} g(y) f(y)\,dy$ provided the integrals exist.

common error Get $E(Y) = 1$ because y is forgotten.

49) * Still have $V(Y) = E(Y-EY)^2 = E(Y^2) - [E(Y)]^2$.

4d) common E1 problem) Given $f(y)$ or $f(y) = c\, g(y)$, find $E(Y)$, $E(Y^2)$ and $V(Y) = E(Y^2) - [E(Y)]^2$.

See HW4 and Q4.

end exam 1 material

begin exam 2 material

42) * p 28 Suppose independent trials, each with $P(\text{success}) = p$, are done until a success occurs. Let X be the number of trials required until the 1st success. Then X has a geometric(p) dist with pmf

$$p(x) = P(X=x) = (1-p)^{x-1} p, \quad x = 1, 2, \ldots$$

need $x-1$ failures if x is the 1st success.

43) p29 Suppose $W \sim bin(n, p)$ where n is large and p is small so that $np \le 7$. Then a $Pois(\lambda = np)$ RV Y is a good approx to W in that $p_W(w) = \binom{n}{w} p^w (1-p)^{n-w} \approx \frac{\lambda^w e^{-\lambda}}{w!} = p_Y(w)$.

44) p29 Let $Y \sim pois(\lambda)$. Then $\sum_{y=0}^{\infty} p(y) = e^{-\lambda} \underbrace{\sum_{y=0}^{\infty} \frac{\lambda^y}{y!}}_{} = e^{-\lambda} e^{\lambda} = 1$.

So $p(y)$ is a pmf.

Taylor (or Maclaurin) series expansion for e^x, $x = \lambda$.

45) Geometric Series: Let $|r| < 1$ and $M \ge 1$ an integer.

Then $\sum_{i=0}^{\infty} r^i = \frac{1}{1-r}$, $\sum_{i=1}^{\infty} r^i = \frac{r}{1-r}$, $\sum_{i=0}^{M} r^i = \frac{1-r^{M+1}}{1-r}$.

If X is geom(p), then $\sum_{x=1}^{\infty} p(x) = p \sum_{x=1}^{\infty} (1-p)^{x-1} =$

$\frac{p}{1-p} \sum_{x=1}^{\infty} (1-p)^x \underset{r=1-p}{=} \frac{p}{1-p} \frac{1-p}{p} = 1$. So $p(x)$ is a pmf.

46) Some texts say $Y = X-1$ is geom(p) with $E(Y) = EX - 1 = \frac{1}{p} - 1 = \frac{1-p}{p}$.

47) The constants that determine a pdf or pmf are called the <u>parameters</u> of the dist.

ex dist parameters

bin(n,p) n, p (21.5)

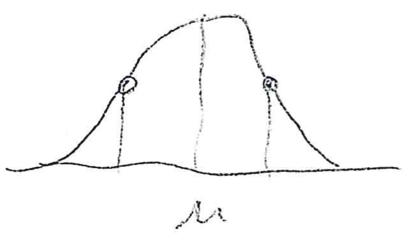
poisson(λ) λ

uniform(θ_1, θ_2) θ_1, θ_2

normal(μ, σ^2) μ, σ^2

48) * p33 * Y has a normal distribution, $Y \sim N(\mu, \sigma^2)$, with parameters $\mu \in \mathbb{R}$ and $\sigma > 0$ if the pdf of Y is $f(y) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}\left(\frac{y-\mu}{\sigma}\right)^2}$, $-\infty < y < \infty$.

49) KNOW If $Y \sim N(\mu, \sigma^2)$, then $E(Y) = \mu$ and $V(Y) = \sigma^2$.



inflection points at $\mu \pm \sigma$

50) If Y is a RV with $EY = \mu$ and $V(Y) = \sigma^2$, then the z score $Z = \frac{Y-\mu}{\sigma}$ has $E(Z) = 0$ and $V(Z) = 1$.

proof] $E(Z) = \frac{1}{\sigma} E(Y-\mu) = \frac{1}{\sigma}(\mu - \mu) = 0$ and

$V(Z) = EZ^2 - 0^2 = E\left(\frac{Y-\mu}{\sigma}\right)^2 = \frac{1}{\sigma^2} E(Y-\mu)^2 = \frac{\sigma^2}{\sigma^2} = 1$.

51) If $Y \sim N(\mu, \sigma^2)$, then $Z = \frac{Y-\mu}{\sigma} \sim N(0,1)$ standard normal.

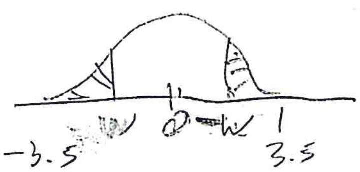
52) Using standard normal table.

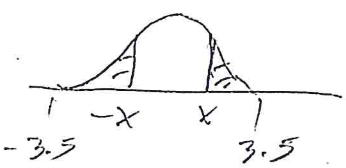
If $Y \sim N(\mu, \sigma^2)$ and $Z \sim N(0,1)$, then

i) $P(Y \geq b) = P\left(Z \geq \frac{b-\mu}{\sigma}\right) = 1 - \overbrace{P\left(Z \leq \frac{b-\mu}{\sigma}\right)}^{\text{table}}$

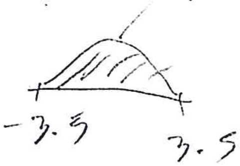
ii) $P(Y \leq a) = P\left(Z \leq \frac{a-\mu}{\sigma}\right)$.

iii) If $w < 0$, $P(Z < w) = P(Z > -w)$





area ≈ 1



iv) If $x > 0$, $P(Z > x) = P(Z < -x)$,

Table gives $P(Z \leq x)$ where $x = z^*$.

$P(Z < -3.5) = P(Z > 3.5) \approx 0$.

53) The standard normal table handout is used to find 3 types of probabilities.

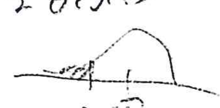
I) a) $P(Z \leq 2.8) = .9974$

z^*	$.00$ ← 2nd digit
2.8	.9974

1st 2 digits

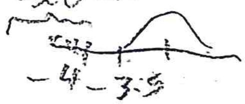
b) $P(Z \leq -1.57) = .0582$

z^*	07
-1.5	.0582

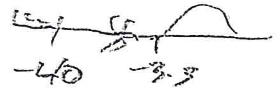


c) $P(Z \leq -4.003) \approx P(Z < -4.00) \approx P(Z < -3.49) \approx 0$

≈ 0 area



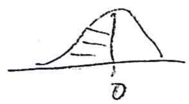
$P(Z < -40) \approx 0$



225

e) $P(Z \le 5) \approx 1 \approx P(Z \le 3.49) \approx 1$

f) $P(Z \le 0) = 0.5$



key words $<, \le$

II) $P(Z \ge z^*) = P(Z > z^*) = 1 - \underbrace{P(Z \le z^*)}_{\text{area from table}}$

a) $P(Z \ge -1) = 1 - P(Z < -1)$ or

z^*	00
-1.0	.1587

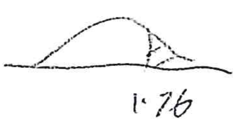
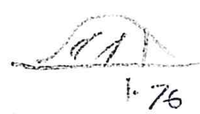
 $= 1 -$  $= 1 - .1587 = \boxed{0.8413}$

b) $P(Z \ge 1.763) \approx P(Z \ge 1.76) = 1 - P(Z < 1.76)$

closest

$= 1 - .9608 = \boxed{0.0392}$

	06
1.7	.9608

 $= 1 -$ 

desired work

	04	05
-1.6	.0505	.0495

c) $P(Z \ge 0) = 0.5$

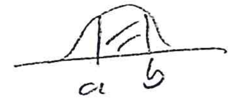
d) $P(Z > -1.645) = 1 - P(Z \le -1.645)$

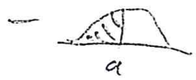
-1.64 and 1.65 are equally close, average the 2 table values

$= 1 - \left(\frac{.0505 + .0495}{2}\right) = 0.95.$ key words $>, \ge$

III)

$= P(a \le z \le b) = P(a \le z < b) = P(z \le b) - P(z \le a)$

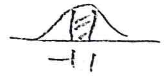
or  = 

− 

	00
−1.0	.1587
1.0	.8413

a) $P(-1 \le z \le 1) = P(z \le 1) - P(z \le -1) =$

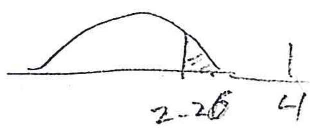
$.8413 - .1587 = \boxed{.6826}$



b) $P(2.257 \le z \le 4.093) \approx P(z \le 4.09) - P(z \le 2.26)$

$\approx 1 - P(z \le 2.26) = 1 - .9881 = \boxed{0.0119}$

	06
2.2	.9881



54) <u>know for E2</u> <u>Formwards calculations!</u>

Finding probabilities when $X \sim N(\mu, \sigma^2)$,

step i) line picture, ii) find z*s iii) normal picture

(iv) use z table

I) $P(X < b) = P(X \le b) = \underbrace{P\left(z \le \frac{b-\mu}{\sigma}\right)}_{\text{table}}$

II) $P(X > a) = P(X \ge a) = P\left(z \ge \frac{a-\mu}{\sigma}\right) = 1 - \overbrace{P\left(z \le \frac{a-\mu}{\sigma}\right)}^{\text{table}}$

III) $P(a \le X \le b) = P\left(\frac{a-\mu}{\sigma} \le Z \le \frac{b-\mu}{\sigma}\right) =$

$\underbrace{P\left(Z \le \frac{b-\mu}{\sigma}\right) - P\left(Z \le \frac{a-\mu}{\sigma}\right)}_{\text{table}}$.

ex] IQ scores $X \simeq N(\mu = 100, \sigma = 15)$

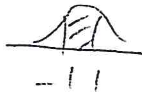
Story problem gives $SD(X) = \sigma$ or $V(X) = \sigma^2$? $\sigma^2 = 225$

a) Find $P(85 < X < 115)$

i) 85 100 115 X $= P(85 < X < 115)$

ii) standardize $\frac{85-100}{15} = -1$, $\frac{115-100}{15} = 1$

	.00
-1.0	.1587
1.0	.8413

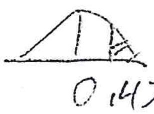
iii)  Z iv) $P(-1 \le Z \le 1) = .8413 - .1587 = \boxed{0.6826}$

b) Find probability that a randomly selected IQ score is greater than 107.

i) 100 107 X

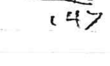
(prob < ½)

ii) $Z = \frac{107-100}{15} \approx \overbrace{0.47}^{\text{table has 2 digits}}$

iii) 

iv) $P(X > 107) = P(Z > 0.47)$

	.07
0.4	.6808

 $= 1 - P(Z < 0.47) = 1 - .6808 = \boxed{0.3192}$