55) common error: [illegible] Then write down table value

eg .6808 instead of .3192
(get 1 − correct answer)

56) Know for E2 — **Backwards calculation**

Finding 100pth percentile x_p such that

$P(X \le x_p) = p$ if $X \sim N(\mu, \sigma^2)$.

i) normal picture — ii) use table inside — iii) unstandardize

always need left tail area

If $P(Z \le z_p) = p$, then $\underbrace{x_p}_{= x^*} = \mu + \sigma \underbrace{z_p}_{= z^*}$.

ex) a) For IQ scores, find the score needed to be in the highest 90%

= lowest 10% = 10th percentile.

i)

4 digit number inside table

i) Find largest prob ≤ 0.1 and smallest prob ≥ 0.1,

Then z^* corresponds to the closer

(if there is a tie, average the z values).

z^*	08	09 (closer)
−1.2	.1003	.0985

so $z^* = -1.28 = \dfrac{x^* - \mu}{\sigma}$

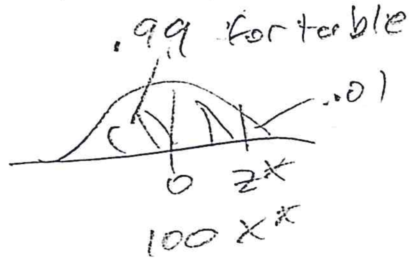
iii) unstandardize: $x^* = \mu + \sigma z^* = 100 + 15(-1.28)$

$= 100 - 19.20 = \boxed{80.80}$

24.5

You can check with a forwards calculation on $P(X \leq 80.8)$,

b) Find the IQ score such that 99% are smaller
= 99th percentile = top 1%.

i) 

ii)

z^*	02	03
2.3	.9898	.9901

closer

so $z^* = 2.33$

iii) unstandardize $x^* = \mu + \sigma z^* = 100 + 2.33(15)$
$= 134.95$ (HW4 R gives a more accurate value.)

Check $P(X \leq 134.95)$

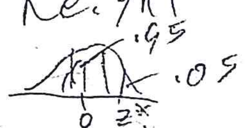
	03
2.3	.9901

$\frac{134.95 - \mu}{\sigma} = \frac{134.95 - 100}{15} = 2.33$

$= .9901$
$= P(X \leq 134.95)$

ex) Suppose 21 year old Belgian F heights ~
$N(\mu = 66 \text{ in}, \sigma = 3 \text{ in})$.

Find height so 5% are taller.

i). 

ii)

z^*	04	05
1.6	.9495	.9505

tie

90% ... $\frac{\quad}{2} = 1.645$

iii) $x^* = \mu + \sigma z^* = 66 + 3(1.645) = 70.935$ in $\approx 5'11$

Note: $1.645 = 95$th percentile and $-1.645 = 5$th percentile if $Z \sim N(0,1)$.

common error: use z^* instead of x^*.

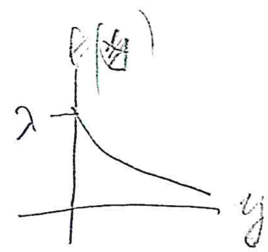
57) Know p32 Y has an exponential distribution,

$Y \sim EXP(\lambda)$, if the pdf of Y is

$f(y) = \lambda e^{-\lambda y}$ for $y > 0$ with $\lambda > 0$.

Then $F(y) = 1 - e^{-\lambda y}$, $y > 0$.

$EY = \frac{1}{\lambda}$ $\quad V(Y) = \frac{1}{\lambda^2}$.



58) p33 The gamma function is

$\Gamma(\alpha) = \int_0^\infty y^{\alpha-1} e^{-y} dy$, $\alpha > 0$.

59) Properties of the gamma function!

i) If $\alpha > 1$, $\Gamma(\alpha) = (\alpha-1)\Gamma(\alpha-1)$ so $\Gamma(\alpha+1) = \alpha\Gamma(\alpha)$.

ii) If n is a positive integer, $\Gamma(n) = (n-1)!$

and $\Gamma(n+\frac{1}{2}) = (n-\frac{1}{2})(n-\frac{3}{2}) \cdots \frac{1}{2}\Gamma(\frac{1}{2})$ with $\Gamma(\frac{1}{2}) = \sqrt{\pi}$.

proof of i) Integration by parts says $\int_a^b u\,dv = uv\Big|_a^b - \int_a^b v\,du$. Assume $b = \infty$ is OK for the gamma fn.

For $\Gamma(\alpha)$, let $u = x^{\alpha-1}$, $dv = e^{-x}dx$

$du = (\alpha-1)x^{\alpha-2}dx$, $v = -e^{-x}$

$$= -x^{\alpha-1} e^{-x} \Big|_0^\infty + (\alpha-1) \underbrace{\int_0^\infty x^{\alpha-2} e^{-x} dx}_{\Gamma(\alpha-1)}$$

25.5

$$= -0 - 0 + (\alpha-1)\Gamma(\alpha-1)$$

since $x^k e^{-x} \to 0$ as $x \to \infty$ for any k (exp fn dominates polynomial).

60] Know p33 Y has a Gamma distr;

$Y \sim G(\alpha, \lambda)$, if the pdf of Y is

$$f(y) = \frac{\lambda^\alpha}{\Gamma(\alpha)} y^{\alpha-1} e^{-\lambda y} \quad \text{where } \alpha, y, \lambda > 0.$$

$$E(X) = \frac{\alpha}{\lambda}, \quad V(X) = \frac{\alpha}{\lambda^2}.$$

61} If $Y \sim EXP(\lambda)$, then $Y \sim Gamma(\alpha = 1, \lambda)$.

62} Many texts take $\lambda = \frac{1}{\beta}$ and say $Y \sim EXP(\beta)$ with $EY = \beta$, $V(Y) = \beta^2$

and $Y \sim Gamma(\alpha, \beta)$ with $EY = \alpha\beta$, $V(Y) = \alpha\beta^2$.

63} Know The support of a RV is $\{y : p(y) > 0\}$ or $\{y : f(y) > 0\}$.

The pdf, pmf and cdf formulas are often given for the support or the support and the boundaries of the

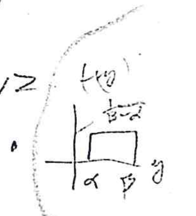
is in an interval
with endpoints a and b where $a = -\infty$ or $b = \infty$ is possible

64) p67 * $Y \sim \chi^2_k$, a <u>chi square dist with k degrees of freedom</u>, if $Y \sim \text{Gamma}\left(\alpha = \frac{k}{2}, \lambda = \frac{1}{2}\right)$.

65) p31 * $Y \sim \text{Uniform}(\alpha, \beta)$ if the support is (α, β)

and $f(y) = \frac{1}{\beta - \alpha}$, $F(y) = \frac{y-\alpha}{\beta-\alpha}$ on the support.

where $\alpha < \beta$. Then $EY = \frac{\alpha+\beta}{2}$, $V(Y) = \frac{(\beta-\alpha)^2}{12}$.



66) * p57 Y has a <u>beta dist</u>, $Y \sim \text{beta}(\alpha, \beta)$,

if $f(y) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} y^{\alpha-1}(1-y)^{\beta-1}$ for $0 < y < 1$.

where $\alpha, \beta > 0$. $E(Y) = \frac{\alpha}{\alpha+\beta}$, $V(Y) = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$,

p215

67) Multivariate prob dist's are used to describe a pop of several variables. Suppose there are n variables $Y_1, \ldots, Y_n$. Then an outcome is $(Y_1 = y_1, \ldots, Y_n = y_n) \equiv (y_1, \ldots, y_n)$.

ex) height, age, weight and gender of SIU students.

68) When $n = 2$, the multivariate prob dist is called a <u>bivariate distribution</u>.

... Y_1 Y_2 be discrete RVs. The (26.5)
<u>joint pmf</u> of Y_1 and Y_2 is

$$p(y_1, y_2) = P(Y_1 = y_1, Y_2 = y_2) \quad \text{for } y_1, y_2 \in \mathbb{R}.$$

ex] 26 in 483 students Y_1 = gender ♀ ♂
Y_2 = major eng noneng
0 1

		Y_2: eng 0	noneng 1
Y_1	F 0	3	5
	M 1	9	9

Randomly select student, $P(Y_1 = 0, Y_2 = 0) = P(F \cap eng)$

$$= \frac{3}{26}.$$

D] A function $p(y_1, y_2)$ is a <u>joint pmf</u> of 2 discrete RVs if

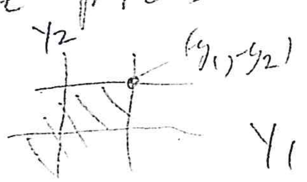
1) $p(y_1, y_2) \geq 0 \quad \forall\, y_1, y_2 \in \mathbb{R}$

2) $\displaystyle\sum_{(y_1, y_2)\,:\, p(y_1, y_2) > 0} p(y_1, y_2) = 1$

D] p42 * A <u>joint cdf</u> for any two RVs Y_1 and Y_2 is

$$F(y_1, y_2) = P(Y_1 \leq y_1, Y_2 \leq y_2), \quad y_1, y_2 \in \mathbb{R},$$

This is the prob of a "southwest corner."



72} A ... the function $f(y_1, y_2)$ is the
joint pdf of Y_1 and Y_2 if

$$F(y_1, y_2) = \int_{-\infty}^{y_2} \int_{-\infty}^{y_1} f(t_1, t_2)\, dt_1\, dt_2 \quad \text{for } y_1, y_2 \in \mathbb{R}$$

73} If $F(y_1, y_2)$ is a joint cdf, then

i) $F(-\infty, \infty) = F(-\infty, y_2) = F(y_1, -\infty) = 0$

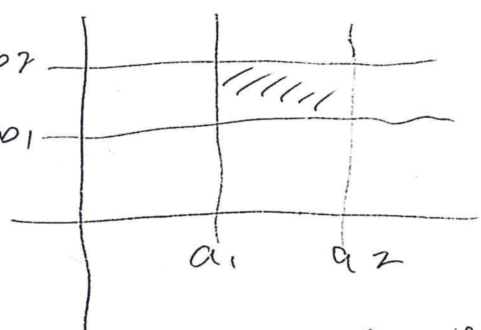
ii) $F(\infty, \infty) = 1$.

74} The function $f(y_1, y_2)$ is a joint pdf if

i) $f(y_1, y_2) \geq 0 \quad \forall\, y_1, y_2 \in \mathbb{R}$

ii) $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(y_1, y_2)\, dy_1\, dy_2 = 1$.

75} * $P(a_1 \leq Y_1 \leq a_2 \overset{\text{intersection}}{;} b_1 \leq Y_2 \leq b_2) = \int_{b_1}^{b_2} \int_{a_1}^{a_2} f(y_1, y_2)\, dy_1\, dy_2$



= volume under surface formed by $f(y_1, y_2)$.

76} A joint pmf for RVs $Y_1, \ldots, Y_n$ is

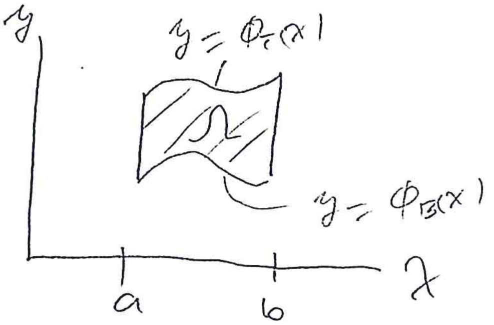
$$p(y_1, \ldots, y_n) = P(Y_1 = y_1, \ldots, Y_n = y_n).$$

77} In the continuous case, $P(Y_1 \leq y_1, \ldots, Y_n \leq y_n)$

$= \int \cdots \int \Phi(\cdots)\, dx = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \underbrace{f(t_1, t_2, \ldots, t_n)}_{\text{joint pdf.}}\, dt_1 \cdots dt_n$

See E2 rev P.3 27.5

18) Evaluation of double integrals with iterated integrals



Ω is the region of integration

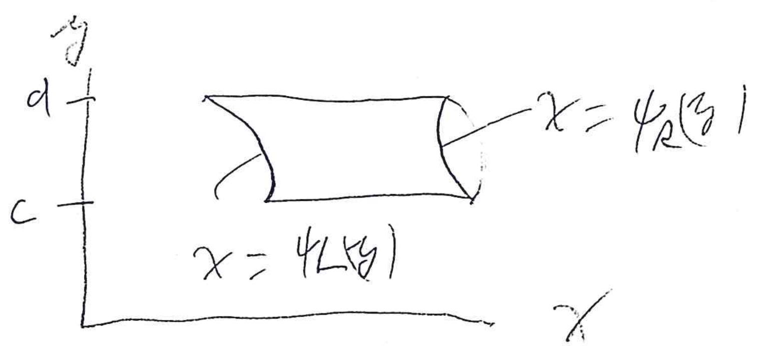
a) Suppose we want $\iint_\Omega f(x,y)\,dx\,dy = \iint_\Omega f(x,y)\,dy\,dx$ where Ω is bounded on the top by the function $y = \Phi_T(x)$, on the bottom by $\Phi_B(x)$, and to the left and right by the lines $x=a$ and $x=b$.

Then $\iint_\Omega f(x,y)\,dx\,dy = \iint_\Omega f(x,y)\,dy\,dx =$

$$\int_a^b \Bigg[\underbrace{\int_{\Phi_B(x)}^{\Phi_T(x)} f(x,y)\,dy}_{\text{treat } x \text{ as a constant}} \Bigg] dx$$

match

b)

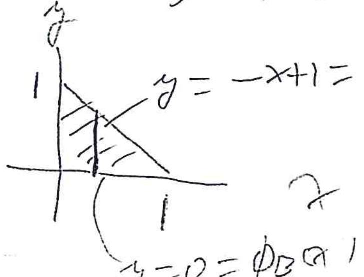


Now suppose Ω is bounded on the left by $\Psi_L(y)$, on the right by $\Psi_R(y)$ and on the top and bottom by the lines $y=d$ and $y=c$. Then

$$\iint_A f(x,y)\,dx\,dy = \int_c^d \Bigg[\underbrace{\int_{\psi_L(y)}^{\psi_R(y)} f(x,y)\,dx}_{\text{treat } y \text{ as a constant.}} \Bigg] dy$$

Ex) Let $f(x,y) = \begin{cases} 24xy & \text{for } \overbrace{0 \le x \le 1,\ 0 \le y \le 1 \text{ and } x+y \le 1}^{\text{replace } \le \text{ by } = \text{ to get lines}} \\ 0 & \text{else} \end{cases}$

Show $\iint f(x,y)\,dx\,dy = 1$ so that $f(x,y)$ is a pdf.

a) 

test points $(0,0)$ and $(1,1)$

make a line parallel to what you are integrating: $dy \parallel y$ axis

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,dx\,dy =$$

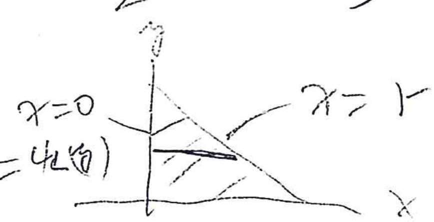
$$\int_0^1 \left[\int_{\phi_B(x)}^{\phi_T(x)} f(x,y)\,dy \right] dx =$$

$$\int_0^1 \left[\int_0^{1-x} 24xy\,dy \right] dx =$$

$$\int_0^1 \left[\frac{24xy^2}{2} \Big|_{y=0}^{y=1-x} \right] dx = \int_0^1 12x(1-x)^2\,dx =$$

$$\int_0^1 12x[1-2x+x^2)\,dx = \int_0^1 12x - 24x^2 + 12x^3\,dx$$

$$= \frac{12x^2}{2} - \frac{24x^3}{3} + \frac{12x^4}{4} \Big|_0^1 = 6 - 8 + 3 = 1$$

b) 

Make line parallel to what you are integrating: $dx \parallel x$ axis

$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,dx\,dy = \int_0^1 \left[\int_{h_2(y)}^{h_1(y)} f(x,y)\,dx\right] dy =$

$\int_0^1 \left[\int_0^{1-y} 24xy\,dx\right] dy = \int_0^1 \left[24y \frac{x^2}{2}\Big|_{x=0}^{x=1-y}\right] dy$

$= \int_0^1 12y(1-y)^2\,dy = 6-8+3=1$ by 9) with

x replaced by y.

p42

79)* If Y_1 and Y_2 have joint pmf $p(y_1, y_2)$,

then the marginal pmf's for Y_1 and Y_2 are

$P_{Y_1}(y_1) = \sum_{y_2} p(y_1, y_2), \forall y_1$ and

hold fixed

$P_{Y_2}(y_2) = \sum_{y_1} p(y_1, y_2) \quad \forall y_2.$

Given a table, ignore these formulas and get the marginal pmf's from the row and column sums.

ex)

		Y_2 eng 0	Y_2 non eng 1	$P_{Y_1}(y_1)$
Y_1	F 0	3/26	5/26	8/26
	M 1	9/26	9/26	18/26
$P_{Y_2}(y_2)$		12/26	14/26	grand total $= 1 = \frac{26}{26}$

$P_{Y_1}(0) = P(F) = p(0,0) + p(0,1) = \frac{3}{26} + \frac{5}{26} = \frac{8}{26}$

$P_{Y_1}(1) = P(M) = p(1,0) + p(1,1) = \frac{9}{26} + \frac{9}{26} = \frac{18}{26}$