— i) If $Y_1 \perp\!\!\!\perp Y_2$, then $\text{cov}(Y_1, Y_2) = 0$ if the expected values exist.

ii) $\text{cov}(Y_1, Y_2) = 0$ does not mean $Y_1 \perp\!\!\!\perp Y_2$.

proof i) $\text{cov}(Y_1, Y_2) = E(Y_1 Y_2) - E(Y_1) E Y_2$

$$\underset{\substack{\uparrow \\ \text{ind}}}{=} EY_1\, EY_2 - EY_1\, EY_2 = 0$$

104) <u>common final problem</u>: Given

$f(y_1, y_2) = k\, g(y_1, y_2)$ on triangular or rectangular support,

a) Find k.

b) Find $E(Y_1)$, $E(Y_2)$ and $E(Y_1 Y_2)$.

c) Find $V(Y_1)$ and $V(Y_2)$

d) Find $\text{cov}(Y_1, Y_2)$,

Typically $f(y_1, y_2) = k\, y_1^i\, y_2^j$ on its support where i and j are small nonnegative integers.

<u>common problem</u>

105) Given $p(y_1, y_2)$, find b), c) and d) from 104).

p48-49

106) Properties of covariance

i) $\text{cov}(X, Y) = \text{cov}(Y, X)$

ii) $\text{cov}(cX, Y) = c\, \text{cov}(X, Y)$

Covariance of a Σ sums = double sum of covariances

iii) $\text{cov}\left(\sum_{i=1}^{n} X_i, \sum_{j=1}^{m} Y_j\right) = \sum_{i=1}^{n} \sum_{j=1}^{m} \text{cov}(X_i, Y_j)$

107) * $V\left(\sum_{i=1}^{n} X_i\right) = cov\left(\sum_{i=1}^{n} X_i, \sum_{j=1}^{n} X_j\right)$ 33.5

$$= \sum_{i=1}^{n} \sum_{j=1}^{n} cov(X_i, X_j) = \sum_{i=1}^{n} \underbrace{cov(X_i, X_i)}_{V(X_i)} + \sum_{i=1}^{n} \sum_{j \neq i} cov(X_i, X_j)$$

$$= \sum_{i=1}^{n} var(X_i) + 2 \sum_{i=1}^{n} \sum_{j<i} cov(X_i, X_j)$$

108) p49 <u>know</u> If $Y_1, \dots, Y_n$ are ind,

$$V\left(\sum_{i=1}^{n} X_i\right) = \sum_{i=1}^{n} V(X_i).$$

109) <u>know</u> $V(aX) = a^2 V(X)$.

110) <u>know</u> The <u>sample mean</u> $\bar{X} = \frac{1}{n} \sum_{i=1}^{n} X_i$

if $X_1, \dots, X_n$ are RVs.

111) p49-50 <u>know</u> If $X_1, \dots, X_n$ are <u>independent and identically distributed</u> (iid) with $EX_i = \mu$ and $V(X_i) = \sigma^2$, then

i) $E(\bar{X}) = \mu$

ii) $V(\bar{X}) = \frac{\sigma^2}{n}$.

proof: $E(\bar{X}) = E \frac{1}{n} \sum_{i=1}^{n} X_i = \frac{1}{n} \sum_{i=1}^{n} E(X_i) = \frac{1}{n} \sum_{i=1}^{n} \mu = \frac{1}{n} n\mu = \mu$

$V(\bar{X}) = V\left(\frac{1}{n} \sum_{i=1}^{n} X_i\right) = \frac{1}{n^2} V\left(\sum_{i=1}^{n} X_i\right) \overset{ind}{=} \frac{1}{n^2} \sum_{i=1}^{n} V(X_i) = \frac{1}{n^2} n\sigma^2 = \frac{\sigma^2}{n}$.

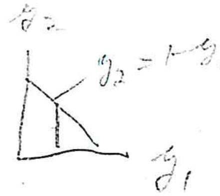
ex) $f(y_1,y_2) = \begin{cases} 24 y_1 y_2 & 0 \le y_1 \le 1,\ 0 \le y_2 \le 1,\ y_1 + y_2 \le 1 \\ 0 & \text{else} \end{cases}$

we showed $f_{Y_1}(y_1) = 12 y_1 (1-y_1)^2 \quad 0 \le y_1 \le 1$

so $EY_1 \overset{= \int y_1 f_{Y_1}(y_1)\, dy_1}{=} \int_0^1 y_1\, 12 y_1 (1-y_1)^2\, dy_1 = \int_0^1 12 y_1^2 (1 - 2y_1 + y_1^2)\, dy_1$

$= \int_0^1 12 (y_1^2 - 2 y_1^3 + y_1^4)\, dy_1 = 12 \left(\frac{y_1^3}{3} - 2\frac{y_1^4}{4} + \frac{y_1^5}{5} \right) \Big|_0^1$

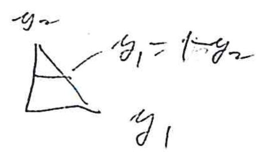
$= 12 \left(\frac{1}{3} - \frac{1}{2} + \frac{1}{5} \right) = 12\, \frac{10 - 15 + 6}{30} = \frac{12}{30} = \frac{2}{5}$



hard way: $EY_1 = \int_0^1 \int_0^{1-y_1} y_1\, 24 y_1 y_2\, dy_2\, dy_1$

$= \int_0^1 \left[\int_0^{1-y_1} 24 y_1^2 y_2\, dy_2 \right] dy_1 =$

$\int_0^1 \left[24 y_1^2 \frac{y_2^2}{2} \Big|_{y_2=0}^{y_2 = 1-y_1} \right] dy_1 = \int_0^1 12 y_1^2 (1-y_1)^2\, dy_1 = \frac{2}{5}$ by above.



harder way: $EY_1 = \int_0^1 \int_0^{1-y_2} y_1\, 24 y_1 y_2\, dy_1\, dy_2$

$= \int_0^1 \int_0^{1-y_2} 24 y_1^2 y_2\, dy_1\, dy_2 = \int_0^1 \left[24 \frac{y_1^3}{3} y_2 \Big|_{y_1=0}^{y_1 = 1-y_2} \right] dy_2$

$= \int_0^1 8 (1-y_2)^3 y_2\, dy_2. = \cdots = \frac{2}{5}$

$EY_1Y_2 = \int_0^1 \int_0^{1-y_1} y_1 y_2\, 24 y_1 y_2\, dy_2\, dy_1 =$

$\int_0^1 \int_0^{1-y_1} 24 y_1^2 y_2^2\, dy_2\, dy_1 = \int_0^1 \left[24 y_1^2 \frac{y_2^3}{3} \Big|_0^{1-y_1} \right] dy_1$

$= \int_0^1 8 y_1^2 (1-y_1)^3\, dy_1 = \int_0^1 8 y_1^2 (1 - 2y_1 + y_1^2)(1-y_1)\, dy_1$

$= \int_0^1 8 y_1^2 (1 - 2y_1 + y_1^2 - y_1 + 2y_1^2 - y_1^3)\, dy_1 =$

$\ldots 8 \int_0^1 (y_1^2 - 3y_1^3 + 3y_1^4 - y_1^5)\, dy_1$

$= 8\left(\frac{y_1^3}{3} - \frac{3y_1^4}{4} + \frac{3y_1^5}{5} - \frac{y_1^6}{6}\right)\Big|_0^1 =$ $\boxed{34.5}$

$8\left(\frac{1}{3} - \frac{3}{4} + \frac{3}{5} - \frac{1}{6}\right) = 8 \; \frac{20-45+36-10}{60} = \frac{8}{60} = \frac{2}{15}$

$= EY_1Y_2$

so $Cov(Y_1, Y_2) = E(Y_1Y_2) - E(Y_1)E(Y_2)$

$= \frac{2}{15} - \frac{2}{5}\frac{2}{5} = \frac{10-12}{75} = \boxed{\frac{-2}{75}}$

ex)

		Y_2: 0	100	200	
Y_1	100	.2	.1	.2	.5
	250	.05	.15	.3	.5
		.25	.25	.5	1.0

$E(Y_1) = 100(.5) + 250(.5) = \frac{350}{2} = \boxed{175}$

$E(Y_2) = 0(.25) + 100(.25) + 200(.5) = \boxed{125}$

$E(Y_1Y_2) = \sum\sum y_1 y_2 P(y_1, y_2) =$

$100(0)(.2) + 100(100)(.1) + 100(200)(.2)$

$+ 250(0)(.05) + 250(100)(.15) + 250(200)(.3) =$

$0 + 1000 + 4000 +$

$0 + 3750 + 15000 = 23750.$

$Cov(Y_1, Y_2) = E(Y_1Y_2) - E(Y_1)E(Y_2) =$

$23750 - (175)(125) = 23750 - 21875 = \boxed{1875}$

$E(Y_1 - Y_2) = E(Y_1) - E(Y_2) = 175 - 125 = \boxed{50}$

112] If X [illegible]

support $\mathcal{X}$ and $Y = t(X)$, then the pmf of Y is $P_Y(y) = \sum_{x \in \mathcal{X} \mid t(x) = y} P_X(x)$.

113] Know for E2: Given a table for $P_X(x)$, compute $y = t(x)$, and collect terms.

ex] $y = t(x) = x^2$

$y = x^2$	1	0	1
x	-1	0	1
$P_X(x)$	.1	.4	.5

Find pmf of $Y = X^2$?

y	0	1
$P_Y(y)$	.4	.6

114] t is increasing $\uparrow$ if $y_1 < y_2 \Rightarrow t(y_2) > t(y_1)$

decreasing $\downarrow$ $\quad t(y_2) < t(y_1)$

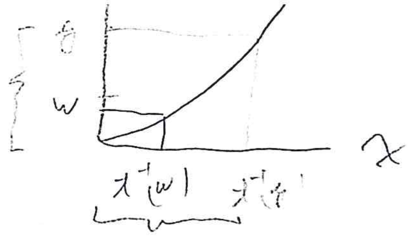
monotone if t is increasing or decreasing

115] * Method of Distributions! Let X and $Y = t(X)$ have pdfs. Let $\mathcal{X} = \{x \mid f_X(x) > 0\}$ and

$\mathcal{Y} = \{y \mid y = t(x) \text{ for some } x \in \mathcal{X}\}$.

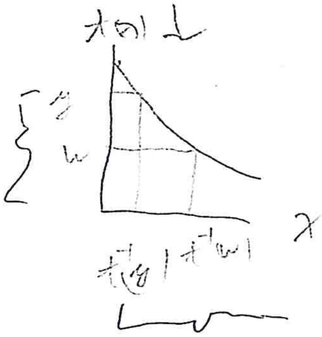
a) If $t \uparrow$ on $\mathcal{X}$, then $F_Y(y) = F_X(t^{-1}(y))$ for $y \in \mathcal{Y}$.

b) If $t \downarrow$ on $\mathcal{X}$, then $F_Y(y) = 1 - F_X(t^{-1}(y))$ for $y \in \mathcal{Y}$



$\{w \in \mathcal{Y} \mid w \le y\} = \{x \in \mathcal{X} \mid t(x) \le y\}$ $\boxed{35.5}$

$= \{x \in \mathcal{X} \mid x \le t^{-1}(y)\}$



$\{w \in \mathcal{Y} \mid w \le y\} = \{x \in \mathcal{X} \mid t(x) \le y\}$

$= \{x \in \mathcal{X} \mid x \ge t^{-1}(y)\}$

sign change if $t \downarrow$

Proof a) $P(Y \le y) = P(t(X) \le y) = P(X \le t^{-1}(y)) = F_X(t^{-1}(y))$

b) $P(Y \le y) = P(t(X) \le y) = \int_{\{x \mid t(x) \le y\}} f_X(x)\,dx = \int_{\{x : x \ge t^{-1}(y)\}} f_X(x)\,dx$

inequality reverses if $t \downarrow$

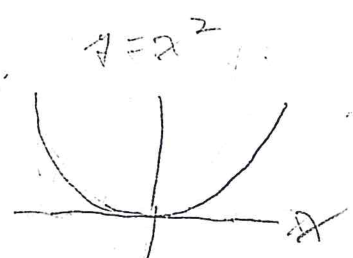
(eg $Y = -X$, $-x \le y \iff x \ge -y = t^{-1}(y)$)

$= \int_{t^{-1}(y)}^{\infty} f_X(x)\,dx = P[X \ge t^{-1}(y)]$

$= 1 - P(X \le t^{-1}(y)) = 1 - F_X[t^{-1}(y)]$.

X contin

11 2) [illegible]



11 $\boxed{3}$ * If $Y = X^2$ has pdf and $\mathcal{X} \subseteq (0,\infty)$, $x^2 \uparrow$

$\subseteq (-\infty, 0)$, $x^2 \downarrow$.

If $Y = X^2$ and $\mathcal{X} = (-a, a)$, $a = \infty$ possible,

then $F_Y(y) = P(Y \le y) = P(X^2 \le y) = P(-\sqrt{y} \le X \le \sqrt{y})$

$= \int_{-\sqrt{y}}^{\sqrt{y}} f_X(x)\,dx = F_X(\sqrt{y}) - F_X(-\sqrt{y})$, $0 \le y \le a^2$.

See p57 with $n=1$. Then $f_Y(y) = \frac{1}{2\sqrt{y}}[f_X(\sqrt{y}) + f_X(-\sqrt{y})]$

117] Know for E2] <u>Method of Transformations for pdfs.</u>

Let X have pdf $f_X(x)$ and support $\mathcal{X}$. Let $Y = t(X)$ where $t\uparrow$ or $t\downarrow$. Suppose $t^{-1}(y)$ has continuous derivative on $\mathcal{Y}$. Then

$$(*)\quad f_Y(y) = f_X(t^{-1}(y)) \left| \frac{d\,t^{-1}(y)}{dy} \right| \quad \text{for } y \in \mathcal{Y}.$$

proof] Use chain rule on $F_Y(y)$ from 115].

$$\frac{d}{dy} F_X(t^{-1}(y)) = f_X(t^{-1}(y)) \frac{d\,t^{-1}(y)}{dy} \quad \text{if } t\uparrow$$

$$\frac{d}{dy} 1 - F_X(t^{-1}(y)) = -f_X(t^{-1}(y)) \frac{d\,t^{-1}(y)}{dy} = f_X(t^{-1}(y)) \left(-\frac{d\,t^{-1}(y)}{dy}\right) \text{ if } t\downarrow.$$

Now $\left| \frac{d\,t^{-1}(y)}{dy} \right| = \begin{cases} \frac{d\,t^{-1}(y)}{dy}, & t\uparrow \\ -\frac{d\,t^{-1}(y)}{dy}, & t\downarrow. \end{cases}$

118] Tips: i) Simplify $(*)$ as much as possible.

ii) To find $x = t^{-1}(y)$, solve $y = t(x)$ for x.

iii) Usually $\mathcal{X}$ is an interval with endpoints $a < b$ ($a = -\infty$, $b = \infty$ possible). Let $t(a) = \lim_{x \downarrow a} t(x)$ and

$x > b$ ... with

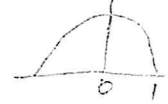
endpoints $t(a)$ and $t(b)$: $(\min(t(a), t(b)), \max(t(a), t(b)))$
[ ]

iv) often Y is gamma or exponential,

v) $Y = \log(X)$ and $Y = e^X$ are common.

ex) $f_X(x) = 3x^2, \; 0 < x < 1$. Find the pdf of $Y = 1 - x^2$.

$y = 1 - x^2 \downarrow$ on $(0,1)$



sol) step i) $y = t(x) = 1 - x^2$; $t(0) = 1$, $t(1) = 0$ so $\mathcal{Y} = (0,1)$.

ii) $y = 1 - x^2$ or $x^2 = 1 - y$ or $x = \sqrt{1-y} = t^{-1}(y) = (1-y)^{\frac{1}{2}}$.

iii) $\left|\dfrac{d\,t^{-1}(y)}{dy}\right| = \left|\frac{1}{2}(1-y)^{-\frac{1}{2}}(-1)\right| = \dfrac{1}{2\sqrt{1-y}}$ on $\mathcal{Y}$

iv) $f_Y(y) = f_X(t^{-1}(y)) \left|\dfrac{d\,t^{-1}(y)}{dy}\right| =$

$$3\left(\sqrt{1-y}\right)^2 \frac{1}{2\sqrt{1-y}} = \underbrace{\frac{3}{2}\sqrt{1-y}, \quad 0 < y < 1}_{f_Y(y) \text{ and the support need to be correct}}$$

§ 2.6 119) The kth moment of Y is $E[Y^k]$.

120) know p58 The moment generating function mgf for a RV Y is $m(t) = \phi(t) = E[e^{tY}]$. The mgf exists

If $\phi(t)$ is finite for $|t| \le b$ for some $b > 0$

Note $\phi(t) = E[g(Y)]$ with $g(y) = e^{ty}$.

Some mgfs of brand name RVS are given on p 1-2 of El rev.

121] Know The probability generating function pgf of a RV Y is $P_Y(z) = E(z^Y)$. The pgf exists if the expectation exists for $z \in (1-\varepsilon, 1+\varepsilon)$ for some $\varepsilon > 0$.

122] The characteristic function of a RV Y is $c(t) = E[e^{itY}]$ where the complex number $i = \sqrt{-1}$. (Note $i^2 = -1$, $i^3 = -i$, $i^4 = 1$

$i^{4k-3} = i$, $i^{4k-2} = -1$, $i^{4k-1} = -i$, $i^{4k} = 1$ for $k = 1, 2, \ldots$)

123] Know $\frac{d^k}{dt^k} \phi_Y(0) = \phi_Y^{(k)}(t)\Big|_0 = E(Y^k)$

$\frac{d^k}{dz^k} P_Y(1) = P_Y^{(k)}(z)\Big|_1 = E[\underbrace{Y(Y-1)\cdots(Y-k+1)}_{k \text{ terms in the product}}]$

So $\phi_Y'(0) = E(Y)$, $\phi_Y''(0) = E(Y^2)$, $P_Y'(1) = E(Y)$,

$P_Y''(1) = E(Y^2) - E(Y)$

independent with mgf $\phi_{X_i}(t)$ and pgf $P_{X_i}(z)$. Then the mgf of S_n is $\phi_{S_n}(t) = \prod_{i=1}^{n} \phi_{X_i}(t)$ and the pgf of S_n is $P_{S_n}(z) = \prod_{i=1}^{n} P_{X_i}(z) = P_{X_1}(z) \cdots P_{X_n}(z)$.

Tips: i) Anything in the product that does not depend on i is treated as a constant.

ii) $\exp(a) = e^a$

iii) $\prod_{i=1}^{n} a^{b\theta_i} = a^{b \sum_{i=1}^{n} \theta_i}$

$$\prod_{i=1}^{n} \exp(b\theta_i) = \exp\left(b \sum_{i=1}^{n} \theta_i\right) = e^{b \sum_{i=1}^{n} \theta_i}.$$

iv) $\sum_{i=1}^{n} a = na$ and v) $\prod_{i=1}^{n} a = a^n$.

125] Know When the mgf and pgf exist, they determine the distribution. So if $\sum_{i=1}^{n} X_i$ has the binomial (n, p) mgf or pgf, then $\sum_{i=1}^{n} X_i \sim \text{bin}(n, p)$.

126] Know Assume the X_i are ind.

a) If $X_i \sim N(\mu_i, \sigma_i^2)$, then $\sum_{i=1}^{n} X_i \sim N\left(\sum_{i=1}^{n} \mu_i, \sum_{i=1}^{n} \sigma_i^2\right)$.

then $\sum_{i=1}^{n} X_i \sim G\left(\sum_{i=1}^{n} \alpha_i, \lambda\right)$

c) $X_i \sim EXP(\lambda) \sim G(1, \lambda)$

so $\sum_{i=1}^{n} X_i \sim G(n, \lambda)$

d) $X_i \sim \chi^2_{k_i} \sim G\left(\frac{k_i}{2}, \frac{1}{2}\right)$

so $\sum X_i \sim \chi^2_{\sum_{i=1}^{n} k_i}$

e) $X_i \sim \text{Poisson}(\lambda_i)$

so $\sum_{i=1}^{n} X_i \sim \text{Poisson}\left(\sum_{i=1}^{n} \lambda_i\right)$

f) $X_i \sim \text{bin}(k_i, p)$ same p (binomial)

so $\sum_{i=1}^{n} X_i \sim \text{bin}\left(\sum_{i=1}^{n} k_i, p\right)$.

g) $X_i \sim NB(k_i, p)$ same p

so $\sum_{i=1}^{n} X_i \sim NB\left(\sum_{i=1}^{n} k_i, p\right)$

h) $X_i \sim \text{geometric}(p) \sim NB(1, p)$

so $\sum_{i=1}^{n} X_i \sim NB(n, p)$.

(27) X has a negative binomial distribution, $X \sim NB(k, p)$, if the pmf of X is $p(x) = \binom{x-1}{k-1} p^k (1-p)^{x-k}$, $x = k, k+1, k+2 \ldots$

Take $p(k) = p^k$. Then $E(X) = \frac{k}{p}$, $V(X) = \frac{k(1-p)}{p^2}$ and

$\phi(t) = \left[\frac{pe^t}{1-(1-p)e^t}\right]^k$. If $X \sim geom(p)$ then $X \sim NB(k=1, p)$

---

proofs for 124 3, 126 a):

Proof 124) $\phi_{\Sigma X_i}(t) = E(e^{t\Sigma X_i}) = E[e^{tX_1 + \cdots + tX_n}] =$

$E[e^{tX_1} e^{tX_2} \cdots e^{tX_n}] \underset{ind}{=} E(e^{tX_1}) E(e^{tX_2}) \cdots E(e^{tX_n})$

$= \prod_{i=1}^{n} \phi_{X_i}(t).$

proof 126 a) If $X \sim N(\mu, \sigma^2)$, then $\phi_X(t) = \exp(t\mu + \frac{t^2}{2}\sigma^2)$.

$X_i \overset{ind}{\sim} N(\mu_i, \sigma_i^2)$

so $\phi_{b_i X_i}(t) = E(e^{t b_i X_i}) = \phi_{X_i}(t b_i)$

$= \exp\left(t b_i \mu_i + \frac{t^2 b_i^2}{2} \sigma_i^2\right)$

so $\phi_{\Sigma b_i X_i}(t) = \prod_{i=1}^{n} \phi_{b_i X_i}(t) = \prod_{i=1}^{n} \exp\left(t b_i \mu_i + \frac{t^2 b_i^2}{2} \sigma_i^2\right)$

$= \exp\left(t \sum_{i=1}^{n} b_i \mu_i + \frac{t^2}{2} \Sigma b_i^2 \sigma_i^2\right) = \exp\left(t \mu_* + \frac{t^2}{2} \sigma_*^2\right)$

the $N(\mu^*, \sigma_*^2)$ mgf. Thus

$\sum_{i=1}^{n} b_i X_i \sim N\left(\sum_{i=1}^{n} b_i \mu_i, \sum_{i=1}^{n} b_i^2 \sigma_i^2\right).$

128) Common problem

i) Given $\phi_X(t)$ or $P_X(z)$, use 123)

to find EX, EX^2, $E(X^2)-E(X)$ and $V(X)=E(X^2)-(EX)^2$

ii) Given a table for $\overset{pmf}{P_X(x)}$, find

$$\phi_X(t) = \sum_x e^{tx} P_X(x) \quad \text{or} \quad \underset{pgf}{P_X(z)} = \sum_x z^x \underset{pmf}{P_X(x)}$$

iii) Given ϕ_X or P_X as in ii), find the pmf $P_X(x)$.

iv) Given a brand name ϕ_X, find the parameters of the brand name RV X.

ex3

y	-1	0	1
$P(y)$	0.3	$.1$	$.6$

$$\phi(t) = E(e^{tY}) = \sum_y e^{ty} P(y) = \underset{P(-1)}{0.3e^{-t}} + \underset{P(0)}{\overbrace{0.1}^{0.1e^{+0t}}} + \underset{P(1)}{0.6\overbrace{e^{t}}^{e^{1\cdot t}}}$$

$$P(z) = E(z^Y) = \sum_y z^y P(y) = 0.3z^{-1} + \underbrace{0.1}_{0.1z^0} + 0.6\underbrace{z}_{z^1}$$

$$\phi'(t) = 0.3e^{-t}(-1) + 0 + 0.6e^{t}$$

$$\phi'(0) = 0.3(-1) + 0 + 0.6(1) = E(Y) = 0.3$$

$$P'(z) = 0.3(-z^{-2}) + 0 + 0.6, \quad P'(1) = 0.3(-1) + 0 + 0.6 = 0.3 = EY$$

$\phi''(t) = -0.3\,e^{-t}(-1) + 0 + 0.6\,e^{t}$ (39.3

$\phi''(0) = 0.3(1) + 0 + 0.6 = \sum y^2 p(y) = EY^2 = 0.9$

$P''(z) = 0.3\left(2z^{-3}\right) + 0 + 0$

$P''(1) = 0.6 = EY^2 - EY = 0.9 - 0.3$

ex) Suppose $\phi_X(t) = e^{5t + 10t^2}$.

Then $X \sim N(5, 20)$.

normal (μ, σ^2) mgf $\downarrow$ $e^{\mu t + \frac{\sigma^2}{2} t^2}$

$\mu = 5$, $\frac{\sigma^2}{2} = 10$

Also $\phi'(t) = e^{5t + 10t^2}(5 + 20t)$

$\phi'(0) = 5e^0 = 5 = E(X)$

$\phi''(t) = (5 + 20t)\, e^{5t+10t^2}(5 + 20t) + 20\, e^{5t+10t^2}$

$= (5 + 20t)^2 e^{5t+10t^2} + 20\, e^{5t+10t^2}$

$\phi''(0) = 25 + 20 = 45 = EX^2$

$V(X) = EX^2 - (EX)^2 = 45 - 25 = 20$

129) When the mgf and pgf exist,

$\phi_X(t) = P_X(e^t)$ and $P_X(z) = \phi_X(\log(z))$

$\uparrow$ ln

130) The mgf of $Y = g(X)$ is

$\phi_Y(t) = E(e^{tY}) = E\left[e^{t g(X)}\right] = \sum_x e^{t g(x)} p_X(x)$, X discr

$\int_{-\infty}^{\infty} e^{t g(x)} f(x)\,dx$, X contin