Math 483 HW 10 2023. Due Monday, Oct. 2. Quiz 4 is Friday, Sept. 29. E(Y), V(Y), mgf m(t) for discrete and continuous RVs, find c so f(y) = c g(y) integrates to one, find probabilities given f(y) or F(y). Find f(y) from F(y) and vice verca. Normal table. Gamma and beta RV's. **Two pages problems A)-E)**.

A) 5.19a Suppose that the joint distribution of  $Y_1$  and  $Y_2$  is given by the table below. Find the marginal probability distribution of  $Y_1$ .

			$y_1$	
$p(y_1, y_2)$		0	1	2
	0	1/9	2/9	1/9
$y_2$	1	2/9	2/9	0
	2	1/9	0	0

comment: The column sums will give it. See ex. 5.5 on p. 237.

B) 5.22 In exercise E) (5.4) on HW9, you were given the following joint probability function where

$$Y_1 = \begin{cases} 0, & \text{if child survived} \\ 1, & \text{otherwise} \end{cases} \text{ and } Y_2 = \begin{cases} 0, & \text{if no belt used} \\ 1, & \text{if adult belt used} \\ 2, & \text{if car-seat belt used}. \end{cases}$$

			$y_1$	
$p(y_1, y_2)$		0	1	total
	0	0.38	0.17	0.55
$y_2$	1	0.14	0.02	0.16
	2	0.24	0.05	0.29
	total	0.76	0.24	1.00

a) Give the marginal probability functions for  $Y_1$  and  $Y_2$ .

b) Give the conditional probability function for  $Y_2$  given  $Y_1 = 0$ .

c) What is the probability that the child survived given that the child was in a car–seatbelt?

comment: a) Get the marginals from the column and row sums.

- b) Find for  $Y_2 = 0, 1, \text{ and } 2$ . Use p. 239.
- c) Use p. 239. See ex 5.7.

C) 5.23ab Let  $Y_1$  and  $Y_2$  have joint pdf

$$f(y_1, y_2) = \begin{cases} 3y_1, & \text{if } 0 \le y_2 \le y_1 \le 1\\ 0, & \text{otherwise.} \end{cases}$$

a) Find the marginal density function for  $Y_2$ .

b) For what values of  $y_2$  is the conditional density  $f(y_1|y_2)$  defined?

comment: a) See p. 236 and ex. 5.6.

- b) See p. 241, want  $f_2(y_2) > 0$ . (**Typo in back of book**.)
  - D) 5.26ad Let  $Y_1$  and  $Y_2$  have joint pdf

$$f(y_1, y_2) = \begin{cases} 4y_1y_2, & \text{if } 0 \le y_1 \le 1, \ 0 \le y_2 \le 1\\ 0, & \text{otherwise.} \end{cases}$$

- a) Find the marginal density functions for  $Y_1$  and  $Y_2$ .
- d) Find the conditional density function of  $Y_2$  given  $Y_1 = y_1$ . comment: See above comment.
- E) 5.27acd Let  $Y_1$  and  $Y_2$  have joint pdf

$$f(y_1, y_2) = \begin{cases} 6(1 - y_2), & \text{if } 0 \le y_1 \le y_2 \le 1\\ 0, & \text{otherwise.} \end{cases}$$

- a) Find the marginal density functions for  $Y_1$  and  $Y_2$ .
- c) Find the conditional density function of  $Y_1$  given  $Y_2 = y_2$ .
- d) Find the conditional density function of  $Y_2$  given  $Y_1 = y_1$ .

See above comment. For c) be very careful about the domain of  $y_1$ . For d), be very careful about the domain of  $y_2$ .