Math 483 HW 10 2023. Due Monday, Oct. 2. Quiz 4 is Friday, Sept. 29. E(Y), $\mathrm{V}(\mathrm{Y})$, mgf $\mathrm{m}(\mathrm{t})$ for discrete and continuous RVs, find c so $\mathrm{f}(\mathrm{y})=\mathrm{cg}(\mathrm{y})$ integrates to one, find probabilities given $f(y)$ or $F(y)$. Find $f(y)$ from $F(y)$ and vice verca. Normal table. Gamma and beta RV's. Two pages problems A)-E).
A) 5.19a Suppose that the joint distribution of $Y_{1}$ and $Y_{2}$ is given by the table below. Find the marginal probability distribution of $Y_{1}$.

|  |  | $y_{1}$ |  |  |
| :---: | :---: | :---: | :---: | :---: |
| $p\left(y_{1}, y_{2}\right)$ |  | 0 | 1 | 2 |
| $y_{2}$ | 0 | $1 / 9$ | $2 / 9$ | $1 / 9$ |
|  | 1 | $2 / 9$ | $2 / 9$ | 0 |
|  | 2 | $1 / 9$ | 0 | 0 |

comment: The column sums will give it. See ex. 5.5 on p. 237.
B) 5.22 In exercise E) (5.4) on HW9, you were given the following joint probability function where

$$
Y_{1}=\left\{\begin{array}{cc}
0, & \text { if child survived } \\
1, & \text { otherwise }
\end{array} \text { and } Y_{2}=\left\{\begin{array}{lc}
0, & \text { if no belt used } \\
1, & \text { if adult belt used } \\
2, & \text { if } \text { car }- \text { seat belt used. }
\end{array}\right.\right.
$$

|  |  |  | $y_{1}$ |  |
| :---: | :---: | :---: | :---: | :---: |
| $p\left(y_{1}, y_{2}\right)$ |  | 0 | 1 | total |
| $y_{2}$ | 0 | 0.38 | 0.17 | 0.55 |
|  | 1 | 0.14 | 0.02 | 0.16 |
|  | 2 | 0.24 | 0.05 | 0.29 |
|  | total | 0.76 | 0.24 | 1.00 |

a) Give the marginal probability functions for $Y_{1}$ and $Y_{2}$.
b) Give the conditional probability function for $Y_{2}$ given $Y_{1}=0$.
c) What is the probability that the child survived given that the child was in a carseatbelt?
comment: a) Get the marginals from the column and row sums.
b) Find for $Y_{2}=0,1$, and 2. Use p. 239.
c) Use p. 239. See ex 5.7.
C) 5.23 ab Let $Y_{1}$ and $Y_{2}$ have joint pdf

$$
f\left(y_{1}, y_{2}\right)=\left\{\begin{array}{cc}
3 y_{1}, & \text { if } 0 \leq y_{2} \leq y_{1} \leq 1 \\
0, & \text { otherwise }
\end{array}\right.
$$

a) Find the marginal density function for $Y_{2}$.
b) For what values of $y_{2}$ is the conditional density $f\left(y_{1} \mid y_{2}\right)$ defined?
comment: a) See p. 236 and ex. 5.6.
b) See p. 241, want $f_{2}\left(y_{2}\right)>0$. (Typo in back of book.)
D) 5.26 ad Let $Y_{1}$ and $Y_{2}$ have joint pdf

$$
f\left(y_{1}, y_{2}\right)=\left\{\begin{array}{cc}
4 y_{1} y_{2}, & \text { if } 0 \leq y_{1} \leq 1,0 \leq y_{2} \leq 1 \\
0, & \text { otherwise }
\end{array}\right.
$$

a) Find the marginal density functions for $Y_{1}$ and $Y_{2}$.
d) Find the conditional density function of $Y_{2}$ given $Y_{1}=y_{1}$.
comment: See above comment.
E) 5.27acd Let $Y_{1}$ and $Y_{2}$ have joint pdf

$$
f\left(y_{1}, y_{2}\right)=\left\{\begin{array}{cc}
6\left(1-y_{2}\right), & \text { if } 0 \leq y_{1} \leq y_{2} \leq 1 \\
0, & \text { otherwise }
\end{array}\right.
$$

a) Find the marginal density functions for $Y_{1}$ and $Y_{2}$.
c) Find the conditional density function of $Y_{1}$ given $Y_{2}=y_{2}$.
d) Find the conditional density function of $Y_{2}$ given $Y_{1}=y_{1}$.

See above comment. For c) be very careful about the domain of $y_{1}$. For d), be very careful about the domain of $y_{2}$.

