Math 483 HW 11 2023. Due Thursday, Oct. 5. Exam 2, Thursday, Oct. 12 through section 5.3 (HW10). **Two pages problems A)-F).**

A) 5.45 Suppose that the joint distribution of Y_1 and Y_2 is given by the table below.

$p(y_1, y_2)$		y_1 0	1	2
y_2	0	1/9	2/9	1/9
	1	2/9	2/9	0
	2	1/9	0	0

In HW10 A), the marginal probability function of Y_1 was shown to be binomial with $n = 2$ and $p = 1/3$. Are Y_1 and Y_2 independent? Why?

comment: The support is not a cross product.

B) 5.48 In exercise B) on HW10, you were given the following joint probability function.

$p(y_1, y_2)$		y_1 0	1	total
y_2	0	0.38	0.17	0.55
	1	0.14	0.02	0.16
	2	0.24	0.05	0.29
	total	0.76	0.24	1.00

Are Y_1 and Y_2 independent? Why?

comment: See ex. 5.10 on p. 248. Is the product of the ith row sum with the jth column sum equal to the ij table entry for all ij entries?

C) 5.52 Let Y_1 and Y_2 have joint pdf

$$f(y_1, y_2) = \begin{cases} 4y_1y_2, & \text{if } 0 \le y_1 \le 1,\ 0 \le y_2 \le 1 \\ 0, & \text{otherwise.} \end{cases}$$

Are Y_1 and Y_2 independent? Why?

comment: Try theorem 5.5. See ex 5.13 on p. 250.

D) 5.53 Let Y_1 and Y_2 have joint pdf

$$f(y_1, y_2) = \begin{cases} 6(1 - y_2), & \text{if } 0 \le y_1 \le y_2 \le 1 \\ 0, & \text{otherwise.} \end{cases}$$

Are Y_1 and Y_2 independent? Why?

comment: Is the support a cross product? Or see HW10 Ea) for the marginals.

E) 5.61 Let Y_1 and Y_2 have joint pdf

$$f(y_1, y_2) = \begin{cases} \frac{1}{8} y_1 e^{-(y_1+y_2)/2}, & \text{if } y_1 > 0,\ y_2 > 0 \\ 0, & \text{otherwise.} \end{cases}$$

Are Y_1 and Y_2 independent? Why?

comment: Recall that $\exp(a+b) = \exp(\text{a})\exp(\text{b})$ and apply theorem 5.5.

F) 5.65b Suppose that for $-1 \leq \alpha \leq 1$, Y_1 and Y_2 have joint pdf

$$f(y_1, y_2) = \begin{cases} [1 - \alpha\{(1 - 2e^{-y_1})(1 - 2e^{-y_2})\}]e^{-y_1-y_2}, & \text{if } y_1 \geq 0,\ y_2 \geq 0 \\ 0, & \text{otherwise.} \end{cases}$$

b) Suppose that the marginal distribution of Y_1 is exponential with mean 1. Find the marginal distribution of Y_2.

comment: The marginal distribution of Y_1 is given. Use the symmetry of the problem to get the marginal distribution of Y_2.