

Math 483 HW 12 2023. Due Monday, Oct. 9. Quiz 5 is Friday, Oct 6.  $E(Y)$ ,  $V(Y)$ , mgf  $m(t)$  for discrete and continuous RVs, Normal table. Find the constant  $k$  such that  $\int_{-\infty}^{\infty} k g(y_1, y_2) dy_1 dy_2 = 1$ . Find marginal and conditional probability functions and pdf's. Exam 2 is on Thursday, Oct. 15, through section 5.3. **Two pages, problems A)-E).**

A) 5.72 Suppose that the joint distribution of  $Y_1$  and  $Y_2$  is given by the table below.

$p(y_1, y_2)$		$y_1$		
		0	1	2
$y_2$	0	1/9	2/9	1/9
	1	2/9	2/9	0
	2	1/9	0	0

a) Find  $E(Y_1)$ .

b) Find  $V(Y_1)$ .

c) Find  $E(Y_1 - Y_2)$ .

comment: Find the marginals  $p_1(y_1)$  and  $p_2(y_2)$ . Then find  $E(Y_1)$ ,  $E(Y_1^2)$  and  $E(Y_2)$ .

B) 5.77 Let  $Y_1$  and  $Y_2$  have joint pdf

$$f(y_1, y_2) = \begin{cases} 6(1 - y_2), & \text{if } 0 \leq y_1 \leq y_2 \leq 1 \\ 0, & \text{otherwise.} \end{cases}$$

a) Find  $E(Y_1)$  and  $E(Y_2)$ .

b) Find  $V(Y_1)$  and  $V(Y_2)$ .

c) Find  $E(Y_1 - 3Y_2)$ .

comment: In HW 10, Ea), you showed that

$$f_{Y_1}(y_1) = 3(1 - y_1)^2, \text{ for } 0 \leq y_1 \leq 1$$

and is zero elsewhere. Use this marginal to find  $E(Y_1)$  and  $V(Y_1)$ . You also showed that

$$f_{Y_2}(y_2) = 6y_2(1 - y_2), \text{ for } 0 \leq y_2 \leq 1$$

and is zero elsewhere. Use this marginal to find  $E(Y_2)$  and  $V(Y_2)$ .

C) 5.89 Suppose that the joint distribution of  $Y_1$  and  $Y_2$  is given by the table below. Find  $Cov(Y_1, Y_2)$ .

		$y_1$		
		0	1	2
$y_2$	0	1/9	2/9	1/9
	1	2/9	2/9	0
	2	1/9	0	0

comment: In problem A), you found  $E(Y_1)$  and  $E(Y_2)$ . Now find  $E(Y_1Y_2)$  and plug into the formula.

D) 5.91 Let  $Y_1$  and  $Y_2$  have joint pdf

$$f(y_1, y_2) = \begin{cases} 4y_1y_2, & \text{if } 0 \leq y_1 \leq 1, 0 \leq y_2 \leq 1 \\ 0, & \text{otherwise.} \end{cases}$$

Show that  $Cov(Y_1, Y_2) = 0$ .

comment: The easiest way to do this is to show that  $Y_1$  and  $Y_2$  are independent.

E) 5.92 Let  $Y_1$  and  $Y_2$  have joint pdf

$$f(y_1, y_2) = \begin{cases} 6(1 - y_2), & \text{if } 0 \leq y_1 \leq y_2 \leq 1 \\ 0, & \text{otherwise.} \end{cases}$$

Find  $Cov(Y_1, Y_2)$ . Are  $Y_1$  and  $Y_2$  independent?

comment: In problem B), you found  $E(Y_1)$  and  $E(Y_2)$ . Find  $E(Y_1Y_2)$  and plug into the formula.