Math 483 HW 13 2033. Due Monday, Oct. 16.
Exam 2, Th. Oct. 12. One page, problems A)-C).
A) 5.103 Assume $Y_{1}$ and $Y_{2}$ are random variables withe $E\left(Y_{1}\right)=2, E\left(Y_{2}\right)=-1$, $E\left(Y_{3}\right)=4, V\left(Y_{1}\right)=4, V\left(Y_{2}\right)=6, V\left(Y_{3}\right)=8, \operatorname{Cov}\left(Y_{1}, Y_{2}\right)=1, \operatorname{Cov}\left(Y_{1}, Y_{3}\right)=-1$ and $\operatorname{Cov}\left(Y_{2}, Y_{3}\right)=0$.
i) Find $E\left(3 Y_{1}+4 Y_{2}-6 Y_{3}\right)$.
ii) Find $V\left(3 Y_{1}+4 Y_{2}-6 Y_{3}\right)$.

Comment: Use theorem 5.12ab on p. 271. See ex. 5.25 and ex. 5.26.
B) 5.105 Let $Y_{1}$ and $Y_{2}$ have joint pdf

$$
f\left(y_{1}, y_{2}\right)=\left\{\begin{array}{cc}
4 y_{1} y_{2}, & \text { if } 0 \leq y_{1} \leq 1,0 \leq y_{2} \leq 1 \\
0, & \text { otherwise }
\end{array}\right.
$$

In HW11 C) it was shown that $Y_{1}$ and $Y_{2}$ are independent. Find $V\left(Y_{1}-Y_{2}\right)$.
Comment: Find the marginal pdf $f_{Y_{1}}\left(y_{1}\right)$ of $Y_{1}$ and then find $V\left(Y_{1}\right)$ by finding $E\left(Y_{1}\right)$ and $E\left(Y_{1}^{2}\right)$. By symmetry, $V\left(Y_{2}\right)=V\left(Y_{1}\right)$. Since $Y_{1}$ and $Y_{2}$ are independent, $\operatorname{Cov}\left(Y_{1}, Y_{2}\right)=0$. Use these numbers in theorem 5.12b.
C) 6.1abce Let $Y$ have pdf

$$
f(y)=\left\{\begin{array}{cc}
2(1-y), & \text { if } 0 \leq y \leq 1 \\
0, & \text { otherwise }
\end{array}\right.
$$

a) Find the pdf of $U_{1}=2 Y-1$.
b) Find the pdf of $U_{2}=1-2 Y$.
c) Find the pdf of $U_{3}=Y^{2}$.
e) Find $E(Y)$ and $E\left(Y^{2}\right)$ and then use these quantities to find
i) $E\left(U_{1}\right)$.
ii) $E\left(U_{2}\right)$, and
iii) $E\left(U_{3}\right)$.

Comment: DO NOT FORGET THE SUPPORT. Do NOT use method of transformations. a) See ex. 6.1 on p 298.
b) Be careful of the negative sign. See an example done in class.
c) See p. 304-307 and an example done in class.
e) See p. 170-171. Find $E(Y)$ and $E\left(Y^{2}\right)$.

