Math 483 HW 14 2023. Due Thursday, Oct. 19. HARD HW! **Two pages, problems A)-H).**

A) 6.23: Let Y have pdf

$$f(y) = \begin{cases} 2(1-y), & \text{if } 0 \le y \le 1 \\ 0, & \text{otherwise.} \end{cases}$$

In HW13 C) you used the method of distribution functions to find the pdfs of a), b) and c). Now use the method of transformations to

a) find the pdf of $U_1 = 2Y - 1$.

b) Find the pdf of $U_2 = 1 - 2Y$.

c) Find the pdf of $U_3 = Y^2$.

comment for A)–D): Don't forget the support. Use formula on p. 313. See ex. 6.6 and 6.7. For B) and D) the distribution is probably exponential.

B) 6.26a: If Y has a Weibull pdf with $\alpha > 0$ and $m > 0$, then

$$f(y) = \begin{cases} \frac{1}{\alpha} m y^{m-1} e^{-y^m/\alpha}, & \text{if } y > 0 \\ 0, & \text{otherwise.} \end{cases}$$

Find the pdf of $U = Y^m$.

C) 6.32: Suppose Y has a uniform distribution on the interval [1,5]. Find the pdf of $U = 2Y^2 + 3$ using the method of transformations.

D) 6.34a: If Y has a Rayleigh pdf with $\theta > 0$, then

$$f(y) = \begin{cases} \left(\frac{2y}{\theta}\right) e^{-y^2/\theta}, & \text{if } y > 0 \\ 0, & \text{otherwise.} \end{cases}$$

Find the pdf of $U = Y^2$.

E) 6.40: Suppose that Y_1 and Y_2 are independent, standard normal random variables. Find the pdf of $U = Y_1^2 + Y_2^2$.

comment: Use theorem 6.4 with $\mu_i = 0$, $\sigma_i^2 = 1$ and n = 2.

F) 6.53a: Let $Y_1, Y_2, ..., Y_n$ be independent binomial random variables with n_i trials and probability of success given by p_i for $i = 1, ..., n$.

a) If all of the n_i's are equal to m and all of the p_i's are equal to p, find the distribution of $\sum_{i=1}^n Y_i$.

comment: Get the mgf from the back of the book and use theorem 6.2. See ex. 6.12 and the proof of theorem 6.3.

G) 6.54a: Let $Y_1, Y_2, ..., Y_n$ be independent Poisson random variables with means $\lambda_1, ..., \lambda_n$, respectively. Find the distribution of $\sum_{i=1}^{n} Y_i$.

comment: Get the mgf from the back of the book and use theorem 6.2. See ex. 6.12 and the proof of theorem 6.3.

H) 6.57: Let $Y_1, Y_2, ..., Y_n$ be independent random variables such that each Y_i has a gamma distribution with parameters α_i and β. Prove that $U = Y_1 + \cdots + Y_n$ has a gamma distribution with parameters $\alpha_1 + \cdots + \alpha_n$ and β.

comment: Get the mgf from the back of the book and use theorem 6.2. See ex. 6.12 and the proof of theorem 6.3.