Math 483 HW 15 2023. Due Monday, Oct. 23. **Two pages, problems A)-E).**

Quiz 6 Friday Oct. 20 is on sections 5.4, 5.5, 5.6, 5.7.

Independence and its relationship to the support. $E(g(Y_1, Y_2))$ especially $E(Y_i)$, $E(Y_i^2)$, $V(Y_i)$, and $E(Y_1Y_2)$. $Cov(Y_1, Y_2) = E(Y_1Y_2) - E(Y_1)E(Y_2)$. $E(h(Y_1)g(Y_2)] = E(h(Y_1))E(g(Y_2))$ if Y_1 and Y_2 are independent. Y_i can be discrete or continuous.

A) 7.15ab Suppose that $X_1, ..., X_m$ and $Y_1, ..., Y_n$ are independent random samples, with the variables X_i normally distributed with mean μ_1 and variance σ_1^2 and the variables Y_i normally distributed with mean μ_2 and variance σ_2^2. The difference between the sample means, $\overline{X} - \overline{Y}$, is then a linear combination of $m + n$ normally distributed random variables, and, by Theorem 6.3, is itself normally distributed.

a) Find $E(\overline{X} - \overline{Y})$.

b) Find $V(\overline{X} - \overline{Y})$.

comment: Let $W_1 = \overline{X}$ and $W_2 = \overline{Y}$. Find $E(W_1), V(W_1), E(W_2)$, and $V(W_2)$ using theorem 7.1. Then apply theorem 6.3 with $a_1 = a_2 = 1$, $Y_1 = W_1, Y_2 = W_2$ and $n = 2$.

B) 7.20 a) If U has a χ^2 distribution with ν degrees of freedom, find $E(U)$ and $V(U)$.

b) Using the results of Theorem 7.3, find $E(S^2)$ and $V(S^2)$ where $Y_1, ..., Y_n$ is a random sample from a normal distribution with mean μ and variance σ^2,

comment: a) Get E(U) and V(U) from the back of the book. b) Use theorem 7.3 and the fact that $E(aS^2) = aE(S^2), V(aS^2) = a^2V(S^2)$.

C) 7.42a The fracture strength of tempered glass averages 14 (measured in thousands of pounds per square inch) and has standard deviation 2. What is the probability that the average fracture strength of 100 randomly selected pieces of this glass exceeds 14.5?

comment: $P(\overline{Y} > 14.5)$ where $\mu = 14$, n = 100. Forwards calculation.

D) 7.43 An anthropologist wants to estimate the average height of men from a certain race of people. If the population standard deviation is assumed to be 2.5 inches and if she randomly samples 100 men, find the probability that the difference between the sample mean and the true population mean will not exceed 0.5 inches.

comment: the CLT applies. Let Z be a normal N(0,1) RV. Want

$$P(-0.5 \leq \overline{Y} - \mu \leq 0.5) \approx P(\frac{-0.5}{\sigma/\sqrt{n}} \leq Z \leq \frac{0.5}{\sigma/\sqrt{n}}).$$

Turn over for E)

E) 7.45 Workers employed in a large service industry have an average wage of \$7.00 per hour with a standard deviation of \$0.50. The industry has 64 workers of a certain ethnic group. These workers have an average wage of \$6.90 per hour. (To see if it reasonable to assume that the wage rate of the ethnic group is equivalent to that of a random sample sample of workers from those employed in the service industry, do the following.)

Calculate the probability of obtaining a sample mean less than or equal to \$6.90 per hour.

comment: Assume that n = 64 is large enough for the CLT to apply.