Math 483 HW 16 2023. Due Thursday, Oct. 26. **One page, problems A)-E).**

Quiz 6 on Friday Oct. 20 covers marginal distributions, $E(Y_1), V(Y_1)$, $\text{Cov}(Y_1, Y_2)$ and independence.

A) 7.73 An airline finds that 5 percent of the persons who make reservations on a certain flight do not show up for the flight. If the airline sells 160 tickets for a flight with only 155 seats, what is the probability that a seat will be available for every person holding a reservation and planning to fly?

comment: Let Y be the number of passengers who do not show up. For everyone to have a seat, need Y greater than or equal to 5. Use the normal approximation to the binomial. See ex. 7.11. $(9(1-p)/p = 171 > 160$ but use the approx anyway.)

B) 7.79a A machine is shut down for repairs if a random sample of items selected from the daily output of the machine reveals too many defectives. Suppose that a random sample of 25 items is selected. If the machine produces 10% defectives, find the probability that the sample will contain at least 2 defective using the normal approximation to the binomial.

comment: Use the normal approximation for the binomial even though $(9(1-p)/p = 81 > 25)$.

C) 7.80 The median age of residents of the United States is 31 years. If a survey of 100 randomly selected United States residents is to be taken, what is the approximate probability that at least 60 will be under 31 years of age?

comment: Since the median age is 31, half of the residents in the population are under 31. Use the normal approximation to the binomial. See ex. 7.11.

D) 7.84 Just as the difference between two sample means is normally distributed for large samples, so is the difference between two sample proportions. That is, if $Y_1$ and $Y_2$ are independent binomial random variables with parameters $(n_1, p_1)$ and $(n_2, p_2)$, respectively, then $(Y_1/n_1) - (Y_2/n_2)$ is approximately normally distributed for large values of $n_1$ and $n_2$.

a) Find $E\left(\dfrac{Y_1}{n_1} - \dfrac{Y_2}{n_2}\right)$.

b) Find $V\left(\dfrac{Y_1}{n_1} - \dfrac{Y_2}{n_2}\right)$.

comment: Theorem 6.3 is useful.

E) 7.93 If $Y$ has an exponential distribution with mean $\theta$, show that $U = 2Y/\theta$ has a $\chi^2$ distribution with 2 degrees of freedom.

comment: Use the method of transformations or the method of distribution functions.