Math 483 HW 17 2023. Due Monday, Oct. 30. **One page, problems A)-E).**

Quiz 7 Friday Oct. 27 will cover transformations for continuous and discrete RV's. If $U = Y_1 + ... + Y_n$ is a sum of random variables, you should be able to find the MGF, expected value, and variance of the sum U. Be able to use the CLT (forwards calculations with the sample mean). Sections 5.8, part of 6.3, 6.4, 6.5, part of 7.2, and 7.3.

Exam 3 is Thursday, Nov. 2. Univariate expected values and mgf's; joint, marginal and conditional pdf's and probability functions; independence; expected values of functions of RV's; covariance; expected value and variance of linear functions of RV's, finding the pdf of U = h(Y); finding the probability function of U = h(Y); CLT; and forwards calculations for the sample mean will be emphasized. Sections 1.3, 3.3, 3.9, 4.2, 4.3, 4.9, 5.2, 5.3, 5.4, 5.5, 5.6, 5.7, 5.8, part of 6.3, 6.4, 6.5, part of 7.2, and 7.3 will be emphasized.

Quizzes 6 and 7 are especially important.

A) 8.1 Using the identity

$$(\hat{\theta} - \theta) = [\hat{\theta} - E(\hat{\theta})] + [E(\hat{\theta}) - \theta] = [\hat{\theta} - E(\hat{\theta})] + B(\hat{\theta}),$$

show that

$$MSE(\hat{\theta}) = E[(\hat{\theta} - \theta)^2] = V(\hat{\theta}) + (B(\hat{\theta}))^2.$$

comment: Use the identity. Note that $B(\hat{\theta})$ is a constant.

B) 8.12ab The reading on a voltage meter connected to a test circuit is uniformly distributed over the interval $(\theta, \theta + 1)$, where θ is the true but unknown voltage of the circuit. Suppose that $Y_1, ..., Y_n$ denotes a random sample of such readings.

a) Show that $\overline{Y}$ is a biased estimator of θ, and compute the bias.

b) Find a function of $\overline{Y}$ that is an unbiased estimator of θ.

comment: Find the expected value of a uniform from the back of the book and use the bias formula on p. 393.

C) 8.40a Suppose that the random variable Y is an observation from a normal distribution with unknown mean μ and variance 1. Find a 95% confidence interval for μ.

comment: Use the CI for μ when σ is known and n = 1. See ex. 8.7 on p. 412-3.

D) "Athletes at major colleges graduated, on the whole, at virtually the same rate as other students," according to the NCAA in 1992. Suppose that in a new poll of 500 athletes at major colleges, the number graduating was 268. Give a 98% confidence interval for p, the proportion of athletes at major colleges who graduate.

comment: See ex. 8.2 on p. 401.

E) 8.58 The administrators at a hospital wished to estimate the average number of days required for in-patient treatment of patients between ages 25 and 34. A random sample of 500 hospital patients between these ages produced a mean and standard deviation equal to 5.4 and 3.1 days, respectively. Construct a 95% confidence interval for the mean length of stay for the population of patients from which the sample was drawn.

comment: See ex. 8.7 on p. 412-3.