Math 483 HW 21 2023. Due **Monday, Nov. 27**. **Two pages, problems A)-E).**
HARD HW;
Exam 4: Friday, Dec. 1.

A) 9.80abd Let $Y_1, ..., Y_n$ be a random sample from the Poisson distribution with mean λ.

a) Find the maximum likelihood estimator $\hat{\lambda}$ for λ.

b) Find the expected value and variance of $\hat{\lambda}$.

d) What is the MLE for $P(Y = 0) = e^{-\lambda}$?

comment: a) Assume that $\sum Y_i > 0$. Differentiate the log likelihood. Show that the critical point is unique and that the second derivative evaluated at the critical point is negative. Hence the critical point is the MLE.
b) $E(\overline{Y}) = E(Y_1)$ and $V(\overline{Y}) = V(Y_1)/n$ if $Y_1, ..., Y_n$ are iid. See p. 332.
d) Use the invariance principle. See p. 480.

B) 9.81 Let $Y_1, ..., Y_n$ be a random sample from the exponential distribution with mean θ. Find the MLE of the variance θ^2. (Hint: Recall Example 9.9.)

comment: Find the MLE of θ, then use the invariance principle to find the MLE of θ^2. Again use the log likelihood, show that the critical point is unique and that the second derivative evaluated at the critical point is negative.

C) 9.83 Let $Y_1, ..., Y_n$ be a random sample from the uniform distribution with pdf

$$f(y|\theta) = \frac{1}{2\theta + 1}$$

for $0 \leq y \leq 2\theta + 1$.

a) Find the MLE of θ.

b) Find the MLE of the variance of the underlying distribution.

comment: a) Let the indicator function $I(x \in A) = 1$ if $x \in A$ and $I(x \in A) = 0$, otherwise. Then the likelihood function is $L(\theta) =$

$$\prod_{i=1}^n \frac{1}{2\theta + 1} I(0 \leq y_i \leq 2\theta + 1) = \left(\frac{1}{2\theta + 1}\right)^n I(0 \leq y_{(n)} \leq 2\theta + 1) = \left(\frac{1}{2\theta + 1}\right)^n I(\frac{1}{2}(y_{(n)} - 1) \leq \theta).$$

Graph the likelihood and find the MLE from the graph.
b) $V(Y_1) = (2\theta + 1)^2/12$. Use the invariance principle.

D) 9.84abd MODIFIED Let $Y_1, ..., Y_n$ be a random sample from the gamma distribution with parameters $\alpha = 2$ and $\beta = \theta$ and with pdf

$$f(y|\theta) = \left(\frac{1}{\theta^2}\right) y e^{-y/\theta}$$

for $\theta > 0$ and $y > 0$.

a) i) Find the MLE of θ.
ii) If $n = 3$, $Y_1 = 120$, $Y_2 = 130$ and $Y_3 = 128$, what is the MLE?

b) Find $E(\hat{\theta})$ and $V(\hat{\theta})$.

d) What is the MLE of the variance of Y?

comment: a) i) Use the log likelihood, show that the critical point is unique and that the second derivative evaluated at the critical point is negative. ii) Then plug in $n = 3, y_1 = 120, Y_2 = 130$, and $y_3 = 128$.

b) $E(c\overline{Y}) = cE(Y_1)$ and $V(c\overline{Y}) = c^2 V(Y_1)/n$ if $Y_1, ..., Y_n$ are iid. Find $E(Y_1)$ and $V(Y_1)$ from the back of the book. Find the expected value and variance for sample size n (not for $n = 3$).

d) Find the variance from the back of the book, then use the invariance principle. Find the MLE for sample size n (not for $n = 3$).

E) A physiology student at SIU in 2002 believed that caffeine would cause a potassium measurement from urine to have a population mean different from 0. Measurements were taken from the urine samples of 5 students. Assume that the measurements are a random sample from a normal distribution, that the sample mean of the measurements was 0.0906 and that the sample standard deviation of the measurements was 0.0806. Test the claim that the population mean of the measurements is different from 0. Use $\alpha = 0.05$.