

Math 483 HW 22 2023. Due Monday, Dec. 4 **Two pages, problems A)-G)**.
Quiz 10 MLE, Method of moments, one sample t test, Monday Nov. 27; Exam 4 Friday,
Dec. 1; Quiz 11 Thursday Dec. 7: six tests of hypotheses, 2 sheets of notes, see review.

A) 9.88 Let Y_1, \dots, Y_n be a random sample with pdf

$$f(y) = (\theta + 1)y^\theta$$

for $0 < y < 1$ where $\theta > -1$. Find the MLE of θ .

comment: Differentiate the log likelihood. Show that the critical point is unique and that the second derivative evaluated at the critical point is negative. Hence the critical point is the MLE. Ignore the method of moments estimator.

comment: B), C) and G) are t-tests in that the test statistic is $\frac{\bar{Y} - \mu_0}{S/\sqrt{n}}$, but the z-table is used for B) and C) while the t-table is used for G).

B) In 1992 *PC World* claimed that consumers could save an average of \$900 by buying a PC from a mail order company. A random sample of 35 customers who had recently purchased a PC from a mail order company had mean and standard deviation of \$885 and \$50, respectively. Do a 4 step test for whether the average savings differs from \$900. **Use $\alpha = 0.01$.**

comment: Step i) is state the hypotheses, step ii) is calculate the test statistic,, step iii) is to find the pvalue, step iv) is to state the decision: reject or fail to reject H_0 and then say what this conclusion means in words). Key word “differs” means the alternative is “not equal.” Use $\alpha = 0.01$. Test statistic should be -1.77 .

C) 10.19 The output voltage for a certain electric circuit is specified to be 130. A sample of 40 independent readings on the voltage for this circuit gave a sample mean of 128.6 and a standard deviation of 2.1. Do a 4 step test of whether the mean output voltage is less than 130.

Comment: test statistic is near -4 and off the table, so the pvalue = 0 or 1.

D) 10.21 Shear strength measurements derived from unconfined compression tests for two types of soils give the results tabled below (measurements in tons per square foot).

Soil type I	Soil type II
$n_1 = 30$	$n_2 = 35$
$\bar{y}_1 = 1.65$	$\bar{y}_2 = 1.43$
$s_1 = 0.26$	$s_2 = 0.22$

Do a 4 step test for whether the soils appear to differ in shear strength. Use $\alpha = 0.01$.

comment: This is a two sample test. The key word is “different”. The test statistic is $\frac{\bar{Y}_1 - \bar{Y}_2 - 0}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}}$. Test statistic is 3.65 and off the table. So the pvalue is 0 or 1.

E) According to a 1994 poll, 60% of 1429 adults interviewed were unable to name an elected official whom the admired. Do a 4 step test for whether a majority of adults are unable to name an elected official whom the admire.

comment: The test statistic is $\frac{\hat{p} - p_o}{\sqrt{\frac{p_o(1-p_o)}{n}}}$. The test statistic is near 7.5 and off the table. So the pvalue is 0 or 1.

F) A survey was conducted in 1992 and another survey in 1991. The 1992 survey was based on 1232 questionnaires while the 1991 survey used 1225. The 1992 survey indicated that 71% of colleges reported an increase in applications, compared with 50% in 1991. Do a 4 step test for whether there was a difference in the proportions of colleges reporting an increase in applications for years 1992 (sample 1) and 1991 (sample 2).

comment: The test statistic is $\frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})(\frac{1}{n_1} + \frac{1}{n_2})}}$ where $\hat{p} = \frac{n_1\hat{p}_1 + n_2\hat{p}_2}{n_1 + n_2}$. The test statistic is near 10.6 and off the table. So the pvalue is 0 or 1.

G) 10.63a MODIFIED A chemical process has produced, on average, 800 tons of chemical per day. The daily yields for the past week are 785, 805, 790, 793 and 802 tons. Assume $\bar{Y} = 795$ and $S^2 = 69.5$. Do a 4 step test for whether the average daily yield is less than 800 tons.

comment: you need to use the t-table with $df = 4$ to show that the pvalue > 0.05 .