

Math 483 HW 5 2023. Due Monday, Sept. 11. EXAM 1 is Thursday, Sept. 14. Quiz 2 on Friday, Sept. 8 will include expectations and variance. (So HW 3, 4 and problems C and D from HW 5). **Two pages problems A)-H).**

A) 2.155abc A group of men possesses the three characteristics of being married (A) having a college degree (B) and being a citizen of a specified state (C) with the fractions given by the accompanying Venn diagram. That is, 5% of the men possess all three characteristics, whereas 20% have a college education but are not married and are not citizens of the specified state. One man is chosen at random from the group.

- a) Find the probability that he is married.
- b) Find the probability that he has a college degree and is married.
- c) Find the probability that he is not from the specified state but is married and has a college degree.

comment: The 8 different regions of the Venn diagram are disjoint.  
The numbers in 7 of the 8 regions correspond to the probability of the region.

B) 3.1 When the health department tested private wells in a community for two impurities commonly found in drinking water, it found that 20% of the wells had neither impurity, 40% had impurity A, and 50% had impurity B. (Some wells had both.) If a well is randomly chosen from those in the county, find the probability distribution for  $Y$ , the number of impurities found in the well.

comment: A Venn diagram may be useful (if only to show work).  
Make a table of  $y$  and  $P(Y = y)$ .

C) 3.10 Let  $Y$  be a random variable with  $p(y)$  given by the accompanying table. Find  $E(Y)$ ,  $E(1/Y)$ ,  $E(Y^2 - 1)$  and  $V(Y)$ .

$y$	1	2	3	4
$p(y)$	0.4	0.3	0.2	0.1

comment: See ex.s 3.2, 3.3, and 3.4.

D) 3.22 A single fair die is tossed once. Let  $Y$  be the number facing up. Find the expected value and variance of  $Y$ .

comment: Using th. 3.6 on p. 96 reduces the amount of work.

E) 3.39 A complex electronic system is built with a certain number of backup components in its subsystems. One subsystem has four identical components, each with a probability of 0.2 of failing in less than 1000 hours. The subsystem will operate if any two of the four components are operating. Assume that the components act independently.

- a) Find the probability that exactly two of the four components last longer than 1000 hours.
- b) Find the probability that the subsystem operates longer than 1000 hours.

comment: Let  $Y$  be the number of components that last longer than 1000 hours. Then  $Y$  is  $\text{bin}(n=4, p=0.8)$ . See ex. 3.8 and ex 3.9 on p. 105-6.

F) 3.44c A new surgical procedure is successful with a probability of  $p$ . Assume the operation is performed five times and the results are independent.

- c) What is the probability that less than two are successful if  $p = 0.3$ ?

comment: See ex. 3.8 and ex. 3.9 on p. 105-6.

G) 3.56 An oil exploration firm is formed with enough capital to perform ten explorations. The probability of a particular exploration being successful is 0.1. Assume that the explorations are independent. Find the mean and variance of the number of successful explorations.

comment: Use th. 3.7 on p. 107.

H) 3.69cd A particular concentration of a chemical found in polluted water has been found to be lethal to 20% of the fish that are exposed to the concentration for 24 hours. Twenty fish are placed in a tank containing this concentration of chemical in water.

- c) Find the probability that at most 16 survive.
- d) Find the mean and variance of the number that survive.

comment: Let  $Y$  be the number that survive. Note that the probability  $p$  that a randomly selected fish survives is obtained using the complement rule.

- c)  $P(\text{at most } 16) = 1 - p(17) - p(18) - p(19) - p(20)$ .
- d) Use th. 3.7 on p. 107.