Math 483 HW 6 2023. Due Monday, Sept. 18. EXAM 1 is THURSDAY, Sept. 14. **Two pages A)-J).**

A college degree, calculus and this class should prepare you for the first *actuary exam*. Many people with math or engineering degrees become actuaries. Actuaries assign prices to future risks. Some websites about the actuary profession are http://www.beanactuary.org/ www.casact.org www.soa.org www.actuary.org and www.aspa.org.

Some "Calculus refresher" texts include Adams, Hass and Thompson (1998): *How to Ace Calculus*; Ash and Ash (1993): *The Calculus Tutoring Book*; Ayres: *Shaum's Outline Of Theory and Problems of Differential and Integral Calculus*; Banner (2007): *The Calculus Lifesaver*; Bleau (2002): *Forgotten Calculus*; Downing *Calculus the Easy Way*; Huettenmueller *Business Calculus Demystified*; Kelly *The Complete Idiot's Guide to Calculus*, Ryan (2003): *Calculus for Dummies*; Thompson and Gardner (1998): *Calculus Made Easy*. Ch. 1 of the following URL is useful for counting.

`http://www.math.uiuc.edu/~ash/Discrete.html`

A) 3.66a Suppose that Y is a random variable with a geometric distribution. Show that $\sum_y p(y) = \sum_{y=1}^{\infty} q^{y-1} p = 1$.

comment: $\sum_{y=1}^{\infty} p(y) = pq^{-1} \sum_{y=1}^{\infty} q^y$. Use the geometric series, p. 67 or p. 836.

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B) 3.70a    An oil prospector will drill a succession of holes in a given
area to find a productive well. The probability that he is successful on
a given trial is 0.2. What is the probability that the third hole
drilled is the first to yield a productive well?

comment: See p. 115 and ex. 3.13. Remember that q = 1 - p.

C) 3.82   Refer to problem B). The prospector drills holes until he finds
a productive well. How many holes would the prospector expect to drill?

comment: Find the expected value using p. 116.
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D) 3.121a Let Y denote a Poisson random variable with mean $\lambda = 2$. Find $P(Y = 4)$.

comment: See p. 132.

E) 3.131 The number of knots in a type of wood has a Poisson distribution with an average of 1.5 knots in 10 cubic feet of wood. Find the probability that a 10 cubic block of wood has at most one knot.

comment: "At most one" means p(0) + p(1).

F) 3.146 If Y has a binomial distribution with n trials and probability of success p, then the moment generating function for Y is

$$m(t) = (pe^t + q)^n$$

where $q = 1 - p$. Differentiate the moment generating function to find $E(Y)$ and $E(Y^2)$. Then find $V(Y)$.

comment: Follow the Poisson lambda example done in class. Use p. 139-140.

G) 3.155ab Let $m(t) = (1/6)e^t + (2/6)e^{2t} + (3/6)e^{3t}$. Find the following:
a) $E(Y)$.
b) $V(Y)$.

comment: Use p. 139-140.

H) 4.8abd Suppose that Y has density function

$$f(y) = \begin{cases} k\ y(1-y), & \text{if } 0 \le y \le 1 \\ 0, & \text{otherwise.} \end{cases}$$

a) Find the value of k that makes $f(y)$ a probability density function.
b) Find $P(0.4 \le Y \le 1)$.
d) Find $P(Y \le 0.4 | Y \le 0.8)$.

comment: Find k so that f integrates to 1, use p. 164 and 52.

I) 4.11abd Suppose that Y has density function

$$f(y) = \begin{cases} c\ y, & \text{if } 0 \le y \le 2 \\ 0, & \text{otherwise.} \end{cases}$$

a) Find the value of c that makes $f(y)$ a probability density function.
b) Find $F(y)$.
d) Use $F(y)$ to find $P(1 \le Y \le 2)$.

comment: a) Find k so that f integrates to 1, use p. 164 and 52. b) Follow ex. 4.3. d) See p. 164.

J) 4.12acd The length of time to failure (in hundreds of hours) for a transistor is a random variable Y with distribution function give by

$$F(y) = \begin{cases} 0, & y < 0 \\ 1 - e^{-y^2}, & y \ge 0 \end{cases}$$

a) Show that $F(y)$ has the properties of a distribution function.
c) Find $f(y)$.
d) Find the probability that the transistor operates at least 200 hours.

comment: a) See p. 160. c) See p. 161. d) Complement rule may help. Since Y is in hundreds of hours, want P(Y greater than or equal to 2).