Math 483 HW 7 2023. Due Thursday, Sept. 21. Two pages problems A)-H).

Quiz 3 is Friday, Sept. 22. Find E(Y) and V(Y) using an MGF. Should have discrete and continuous random variables. In particular, know how to do binomial and poisson computations.

A) 4.20 If Y has density function

$$f(y) = \begin{cases} (1/2)(2-y), & \text{if } 0 \le y \le 2\\ 0, & \text{otherwise,} \end{cases}$$

find the mean and variance of Y.

comment: See p. 170 - 171. Use the short cut formula to find V(Y).

B) 4.21 If Y has density function

$$f(y) = \begin{cases} (3/2)y^2 + y, & \text{if } 0 \le y \le 1\\ 0, & \text{otherwise,} \end{cases}$$

find the mean and variance of Y.

comment: See p. 170 - 171. Use the short cut formula to find V(Y).

C) 4.24 If Y is a continuous random variable with density function f(y), use Theorem 4.5 to prove that $\sigma^2 = V(Y) = E(Y^2) - [E(Y)]^2$.

comment: The proof on p. 96 works.

D) 4.26 If Y is a continuous random variable with mean μ and variance σ^2 and if a and b are constants, use Theorem 4.5 to prove the following.

a) $E(aY + b) = aE(Y) + b = a\mu + b.$

b)
$$V(aY + b) = a^2 V(Y) = a^2 \sigma^2$$
.

comment: For b) let W = aY + b and use a).

E) 4.44 Suppose Y has density function

$$f(y) = \begin{cases} k, & \text{if } -2 \le y \le 2\\ 0, & \text{otherwise.} \end{cases}$$

a) Find the value of k.

b) Obtain the distribution function F(y) of Y.

comment: a) f(y) integrates to one. b) F(y) should be continuous.

F) 4.51 The cycle time for trucks hauling concrete to a highway construction site is uniformly distributed over the interval 50 to 70 minutes. What is the probability that the cycle time exceeds 65 minutes if it is known that the cycle time exceeds 55 minutes?

comment: This is a conditional probability. See p. 52.

G) 4.52 Refer to exercise 4.51. Find the mean and variance of the cycle times for the trucks.

comment: Use p. 176. OVER for H H) 4.58abcde Use Table 4, Appendix III (the table is also located on the front of the book, this is the normal table handout) to find the following probabilities for a standard normal random variable Z:

a) $P(0 \le Z \le 1.2)$ b) $P(-0.9 \le Z \le 0)$ c) $P(0.3 \le Z \le 1.56)$ d) $P(-0.2 \le Z \le 0.2)$ e) $P(-1.56 \le Z \le -0.2)$ comment: See examples done in class and ex. 4.8 on p. 179.